A Screw Representation for Attitude Estimation and Its Application to Legged Locomotion

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Abstract Efficient motion estimation is central to observing and controlling dynamic legged locomotion. This paper considers a screw-theoretic (line-oriented) representation for this context and illustrates this on the attitude estimation subproblem. This is presented as part of an Extended Kalman Filter (EKF) based on inertial (gyroscopic) sensing in which each measurement axis is treated as an zero-pitch, instantaneous screw axis. The implemented solution integrates this to tracks both the orientation and the body screw-axis. In comparison to point-oriented (quaternion) representations, this method is more general, computationally efficient, and provides a more intuitive mechanism for specifying motion constraints, especially for rotary joint motion(s) such as those that at the foot. This technique is demonstrated on a trotting quadrupedal robot at a 250 Hz rate and with drift errors limited to a 5° bound.

1 Introduction

Gait locomotion offers agility and obstacle traversal over diversity of terrains. Dynamically sensing this motion in field is a non-trivial robotics problem with a variety of solutions. As noted in Singh and Waldron (2007), inertial measurements provide a compact, self-contained sensing option for in-field legged locomotion sensing. However, the application of this approach in the legged domain is complicated by impulsive changes resulting from footfall shocks leading to the problems of saturation and drift.

Regardless of the sensing approach selected, a general and efficient description of the motion is required. There is a long history of mathematical models for rigid body pose including rotation matrices, Euler angles, and unit quaternions. A more general approach is to adopt a line-based representation, such as that proposed by Ball (1900). The formulated screw motion has the advantage of not having singularities, a basis in theoretical kinematics, an intuitive representation, and does not have a quadratic operating constraint.

Screw methods are widely applicable to a variety of kinematic analysis problems in robotics with applications ranging from manipulators to knee motion [Roth (1984), Tsai (1999), Wolf and Degani (2006)]. However, their use in the pose estimation has been limited as most estimators represent rotation matrices or quaternions Kraft (2003).

The paper describes an adaption of the screw methods to the attitude estimation problem using an Extended Kalman Filter (EKF) framework on the basis that this framework is the most general and, as noted in Funda and Paul (1990), is relatively computationally efficient (though not as efficient as quaternions due to redundancies in the Plücker coordinates that define the screw-axis). The paper then highlights the estimation model and experimental operation as demonstrated on a dyanamic quadrupedal robot trotting at speeds from 1.5–2.5 m/s.

2 Orientation Representation

The orientation of a body, by definition, is always relative to some other frame (or body). Since there are fundamentally 3 DOF, a common approach is the use of an Euler angle representation whereby motions are subsequent applications of prescribed orientations about the object. For example, roll, then pitch, then yaw, which is the $z-y-x$ set (i.e., $\mathbf{R}(\text{roll}, \text{pitch}, \text{yaw}) = \mathbf{R}^z(\text{roll})\mathbf{R}^y(\text{pitch})\mathbf{R}^x(\text{yaw})$). While this method is simple, it has the disadvantage of the implicit need for intermediate local frames and has singularities at orthogonal angles (i.e., 0° and 90°), leading to a sensing loss of a DOF and "gimbal-lock" effects.

Another alternative is the use of a 4-dimensional quaternion (i.e. Euler parameters), but these have no obvious physical meaning and require a constant transformation of the simple angular velocity measurements. This introduces implementation complexity as multiple sets of quaternions may describe a given orientation.

The screw vector is a geometric element often used kinematic analysis. It provides a compact, intuitive, and robust representation for spatial motion [Roth (1984)]. This notion is an algebratic formulation of Chasles theorem which states that a general displacement of a rigid body can be achieved by a rotation on a unique axis plus a translation parallel to this axis Davidson and Hunt (2004). A screw displacement may be considered to be identical to this with the screw axis as the unique axis of rotation, the twist as the rotation about this axis, and the pitch as the displacement over this joint. This gives the screw, $\$ = (\mathbf{s}, \mathbf{S_o}.\phi, d)$, as eight parameter representation of the with $\mathbf{s} = (s_x, s_y, s_z)$ as the screw axis direction unit vector, $\mathbf{S_o} = (S_{ox}, S_{oy}, S_{oz})$, as A point on $\mathbf{s}$, ϕ as twist, and d as pitch or translation along $\mathbf{s}$.

If we consider just the orientation (i.e., a zero-pitch screw) then the we have four parameters, but three degrees-of-freedom (DOF) as $\mathbf{s}$ is a unit vector.

Another consideration is that only if the changes in orientation are small may they be considered as being commutative [Kane et al. (1983)]. The consequence of this approximation on dynamic legged locomotion estimation is that the sample times need to be fast enough relative to the stride cycle to insure small displacements. The challenge arises for the rapid angular motions of contact. For example, for a contact impulse that yields a 500°/sec pitch excursion, the ideal sample period is less than 10 milliseconds.

A feature of the screw representation is that its reciprocal (wrench) stipulates the constraining forces and moments on the motion. However, implementation of this constraints is non-trivial. Unlike the 1-DOF joints prevalent in link work mechanisms, the motion of a dynamic legged robot is not so directly confined, especially when consider the flight phase. Further, the wrench is a rigid body constraint, which does not easily translate to the highly-compliant nature present in dynamic legged machines.

3 Method

With a series of approximate models for describing particular sections of the gait and a mechanism for switching them, the paper now considers the state estimation problem in order to determine state from multiple measurements.

The use of camera motion alongside inertial sensors has been considered for aerial and ground applications [Corke (2004)]. In previous work, the use of optical flow as a low-frequency complementary measure for aiding high-frequency inertial measurements was explored and found to estimate orientation as long as it was sufficiently initialized. Inertial data are used to determine the potential energy state, which comes from the vertical position.

3.1 State Vector on KOLT

KOLT uses a fuzzy-control architecture as part of an optimal control strategy [Nichol et al. (2004)] requires a rapid estimate of orientation relative to a ground frame for locomotive control in order to orient the legs and compensate for the dynamics of rapid locomotion. Further, for navigation and dynamic compensation functions, the controller also requires a similarly rapid estimate of its linear motion relative to a ground frame. In the case of KOLT cyclic properties are considered as having deterministic values. That is, they are regarded as being known from the control strategy.

As mentioned, the orientation is represented using a screw-axis framework. In particular, it takes advantage of the notion that each spatial motion maybe considered to be the resultant of individual motions about a unique axis. Principally, this formulation uses the simple angular velocities (ω) measured by each gyro to form an instantaneous screw that is combined with the previous screw axis via the screw-triangle to give the final resultant attitude. For compactness, the final screw-axis definition is extended so that the twist is encoded (in radians) in the screw axis. This implies that the final product is not normalized, which means that it will need to be normalized before subsequent computations are performed.

As expected, the full state vector for estimating KOLT single, rigid body equivalent has 18-dimensions. It is organized in the form

$$\mathbf{x} = \left[s_x, s_y, s_z, x, y, z, \omega_x, \omega_y, \omega_z, \dot{x}, \dot{y}, \dot{z}, B_{\omega_x}, B_{\omega_y}, B_{\omega_z}, B_x, B_y, B_z\right]^{\mathsf{T}} \tag{3.1}$$

3.2 Hybrid EKF Estimation Techniques

Estimation is a process for calculating system variables from measurement source(s). The Hybrid EKF is a state-space approach that is optimal in a least-squares sense under the (strict) assumption of white, mutually independent linear environments [Singh and Waldron (2007)]. Alternative estimation algorithms, such as the Unscented KF or Particle filter, provide better linearization; however, their use has to be balanced against computational resources and the update rates required.

Using the notation adopted in previous work Singh and Waldron (2005), we define $\mathbf{x}$ as the target state vector, $\mathbf{F}$ as the system dynamics matrix, $\mathbf{H}$ as the measurement matrix, and $\mathbf{v}$ and $\mathbf{w}$ as the process and measurement noise vectors respectively. Thus, the system can be modeled as $\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{v}$, and the measurement as $\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{w}$. Hybrid models can be implemented as a function that smoothly varies $\mathbf{F}$.

3.3 Test Platform

The goals of this method are to obtain state estimates robust to the eccentricities present in legged locomotion. To do this, the technique was extended to the KOLT robot, where the principal task is to estimate attitude (especially pitch) for use in its trot and galloping controllers. The implemented estimator operates at peak rates of 250 Hz. In practice, this was often run at half the rate to free resources.

The Kinetically Ordered Locomotive Tetrapod robot is a testbed for dynamic legged locomotion theory with application to high-capacity legged robots [Nichol et al. (2004)]. Its four identical 3-DOF legs are fully actuated. Its speed presents a significant challenge as control simulations indicate the need for rapid pitch feedback at rates >50 Hz.

4 Experimental Results

Experiments were performed on KOLT to evaluate the performance of the HEKF method. To facilitate comparison and to ensure safe robot operation, the robot was connected to an instrumented boom arm. The arm is 2.75 *m* long has has 3-DOF (pitch along the axis plus roll and yaw about the center post). For these experiments, data from precision encoders (6,000 count) on the boom arm were considered to be the control values (i.e., arm and KOLT coupling flexion are assumed to be negligible). The large boom arm radius resulted in small pose changes per sample, especially for both yaw. Synchronization of the control was made by having KOLT record boom encoder data.

The experiments were performed for bound and trot gaits that were programed using the symmetric and virtual leg methodologies. As current research is refining a sustained galloping controller, data for the gallop are not presented. To increase resources available for control, the estimator was simplified for KOLT operation as only to track attitude values. Thus, the estimator's state space had of 9-terms (all three DOF for 3D motion, plus their first derivatives, plus their biases).

The results of the HEKF estimator for a typical trotting experiment are shown in Figs. 1 and 2. For comparison with the encoders the estimated state values were transformed to the boom arm origin frame. The gyro covariance and initial bias value was found through a calibration procedure. Many experiments were performed for short periods of time (∼1 minute), yet the inertial drift, if unchecked, would have exceeded practical limits (i.e., greater than 90 deg.).

For dynamic trotting motion, such as that shown in Fig. 2, the HEKF estimator has an error of approximately 5 deg. RMS. This large an error might seem surprising, but can be attributed to errors in the inertial measurements which lead to biases in the estimate. Further, when the inertial data are significantly in error, the HEKF is not able to adapt rapidly to become more reliant on the aided (visual) data. Tuning the HEKF for this would lead to a case where the estimator over weights the importance of the visual data, which will lead to the estimates lagging (due to the delay of the visual measurements). At an extreme, this is equivalent to operating without inertial measurements.

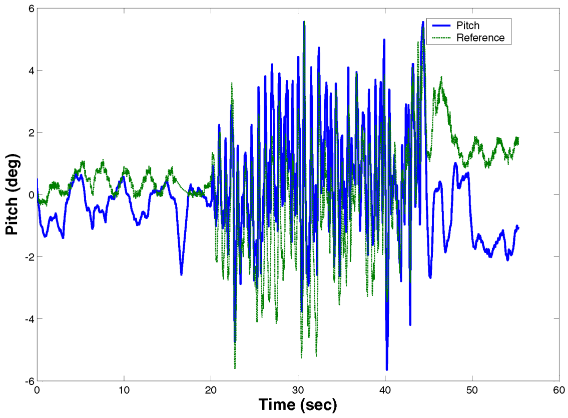

Figure 1. The pitch data from the HEKF inertial estimator and from the reference encoder on the boom arm show stable estimator performance over 36 cycles (during ∼20 seconds of running). The loss of balance, as seen during the final landing, is poorly compensated by the estimator.

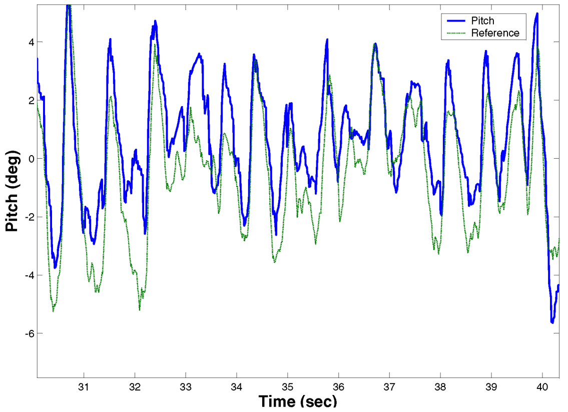

Figure 2. A sub-section of the trotting pitch data. The tracking performance is improved compared to general motion, especially for rapid positive (nose upwards) pitch motions.

5 Conclusions

Due to the discontinuities, dynamic legged locomotion is a unique domain separate from aerial or wheeled vehicles. This is treated using a hybrid model that models the trot as consisting of two dynamic modes and the gallop as three. Modes are transitioned using an energy based metric. This builds on and is consistent with prior work in this field, though adopts a kinetic instead of kinematic framework for hybrid transitions.

As measured by pitch excursion changes, the performance of the estimator on the quadruped is sound with the estimator converging. In fairness, the trot gait is a more stable motion that is compatible with the constant angular velocity (no torque) flight phase assumption.

To an extent, this result is somewhat expected as the hybrid model incorporates more information and is a prudent means of capturing the discrete dynamics in an implementation. The interesting result is this also suggests that an efficient cycle may be constructed with only three instead of the five states suggested by Raibert or the 120 possible transitions present.

The experiment contributes a quantification of KOLT performance. It also shows that modifying HEKF estimation techniques to include characteristics unique to trotting or galloping legged movement results in stable self-contained attitude tracking with low latency and fast updates, which would not be possible with one sensing modality alone.

6 Future Work

The quadruped dynamic gait models are simplistic by design so as to enable faster computation and on-line control application. In off-line analysis applications these constraints are less stringent and hence future work is considering the extension of the impulse methods to yield more complete, yet efficient, solutions. Even with a more involved model, the estimator could act as a smoother instead of a filter, updating the entire state history when resources are free. An additional area under consideration is tweaking the estimator to take advantage of the redundancy in Plücker as a check on errant measurements.

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