The objective of this tutorial is to explore homogenous transformations. The MATLAB robotics toolbox developed by Peter Corke might be a useful aid\(^1\).

**Reading**
Please read/review chapter 7 of Robotics, Vision and Control.

**Review**
Useful commands:
Transl, trotx, troty, trotz, rotx, roty, rotz, tr2eul, DHFactor

Familiarise yourself with the link class

**Questions**
1. For the robot shown in the following figure, find the table of DH parameters according to “Standard” DH conventions.
   (note: you are allowed to move the initial frame to fit convention(s))

   ![Diagram of a robot]

   **Legend:**
   - Revolute Joint - in plane
   - Revolute Joint - out of plane
   - Revolute Joint - about axis
   - Prismatic Joint
   - End Effector

   **Answers:**

<table>
<thead>
<tr>
<th>Link</th>
<th>FromFrame</th>
<th>ToFrame</th>
<th>( \theta_i )</th>
<th>( d_i )</th>
<th>( a_i )</th>
<th>( \alpha_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>( 01^* )</td>
<td>4</td>
<td>1</td>
<td>-90°</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>( 02^* )</td>
<td>0</td>
<td>2</td>
<td>90°</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>( 03^* )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

   ➔ Note that the position of the end effector (the gripper) may be viewed as a position vector (\( \mathbf{P}_{\text{end,effector}} \)) in Frame 3.

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\(^1\) [http://petercorke.com/Robotics_Toolbox.html](http://petercorke.com/Robotics_Toolbox.html)
a.) Determine the joint angles of the two-link planar arm.

The joint space of the robot is \((\theta_1, \theta_2)\).
The forward kinematics may be solved directly using the vector-loop method or somewhat more mechanically using the DH convention (see slides 24 and 42 of Lecture 3). This gives:

\[
(p_x, p_y) = (a_1 \cos \theta_1 + a_2 \cos \theta_{12}, \quad a_1 \sin \theta_1 + a_2 \sin \theta_{12})
\]

The inverse kinematics involves solving the above simultaneous equation for \(\theta_1\) and \(\theta_2\).

A geometric way of solving this is to observe that the distance from \(\{0\}\) to \(\{2\}\) is independent of \(\theta_1\). Thus, sum of squares gives:

\[
p_x^2 + p_y^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \theta_2
\]

\[
\theta_2 = \arccos \left( \frac{p_x^2 + p_y^2 - a_1^2 - a_2^2}{2a_1a_2} \right)
\]

If \(\theta^*\) is an answer to the above, the, in general, \(-\theta^*\) will also be an answer. This is corresponds to the “elbow up” and “elbow down” configurations.

Substituting this back into the kinematic equations gives:

\[
p_x = (a_1 + a_2 \cos \theta_{2}) \cos \theta_1 - (a_2 \sin \theta_2) \sin \theta_1, \quad p_y = (a_2 \sin \theta_2) \cos \theta_1 + (a_1 + a_2 \cos \theta_{2}) \sin \theta_1
\]

\[
c\theta_1 = \frac{p_x (a_1 + a_2 \cos \theta_{2}) + p_y (a_2 \sin \theta_2)}{a_1^2 + a_2^2 + 2a_1a_2 \cos \theta_2}
\]

\[
s\theta_1 = \frac{-p_x (a_2 \sin \theta_2) + p_y (a_1 + a_2 \cos \theta_{2})}{a_1^2 + a_2^2 + 2a_1a_2 \cos \theta_2}
\]

\[\theta_1 = \text{Atan2}(s\theta_1, c\theta_1)\]
If \(a_1 = 2\) and \(a_2 = 3\) what are the joint angles corresponding to an end effector position of \((x,y) = (1, 1)\).

\[\theta_1 = 167.028^\circ, \theta_2 = -156.44^\circ \text{ (Elbow down)}\]

Or \(\theta_1 = -77.028^\circ, \theta_2 = 156.44^\circ \text{ (Elbow up)}\)

To verify using the Robotics Toolbox:

\[
\text{L}(1) = \text{Link}([0 0 2 0], 'standard')
\]

\[
\text{L}(2) = \text{Link}([0 0 3 0], 'standard')
\]

\[
two\text{link} = \text{SerialLink(L, 'name', 'two link')}
\]

\[
T = \text{rpy2tr}(0, 0, 0); T(1:2, 4) = [1 1]
\]

\[
\text{Qsol} = \text{two\text{link}.ikine}(T, \text{zeros}(1, 2), [1 1 0 0 0 0])
\]

\[
\text{Qsol} =
\]

\[
2.9152, -2.7305
\]