METR 4202 / 7202 -- Robotics **Problem Set I**

Kinematics: First Impression!

Objective

Robotics and automation can be true to form. Yet, can a stamp be beyond its mark?

Questions

1. Rotation Matrices Turned Around [16%]

Given the following 3x3 rotation matrices:

$$R_1 = \begin{bmatrix} 0.7071 & -0.7071 & 0 \\ 0.7071 & 0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R_2 = \begin{bmatrix} 0.6124 & -0.5 & 0.6124 \\ 0.3536 & 0.866 & 0.3536 \\ -0.7071 & 0 & 0.7071 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 0.6124 & 0.6124 & 0.5 \\ 0.3536 & 0.3536 & -0.866 \\ -0.7071 & -0.7071 & 0 \end{bmatrix}, R_4 = \begin{bmatrix} 0 & 0.2588 & 0.9659 \\ 0 & 0.9659 & -0.2588 \\ -1 & 0 & 0 \end{bmatrix}$$

$$R_5 = \begin{bmatrix} -0.9122 & 0.4098 & 0 \\ -0.4098 & -0.9122 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$R_7 = \begin{bmatrix} 0.8889 & 0.1111 & 0.6667 \\ 0.3333 & 0.5556 & 0.7778 \\ 0.4444 & 1 & 0.2222 \end{bmatrix}, R_8 = \begin{bmatrix} 0.1191 & -0.9201 & 0.3731 \\ 0.8592 & -0.0927 & -0.5031 \\ 0.4975 & 0.3805 & 0.7795 \end{bmatrix}$$

a. Are these (within practical numerical limits) valid rotation matrices? Why?

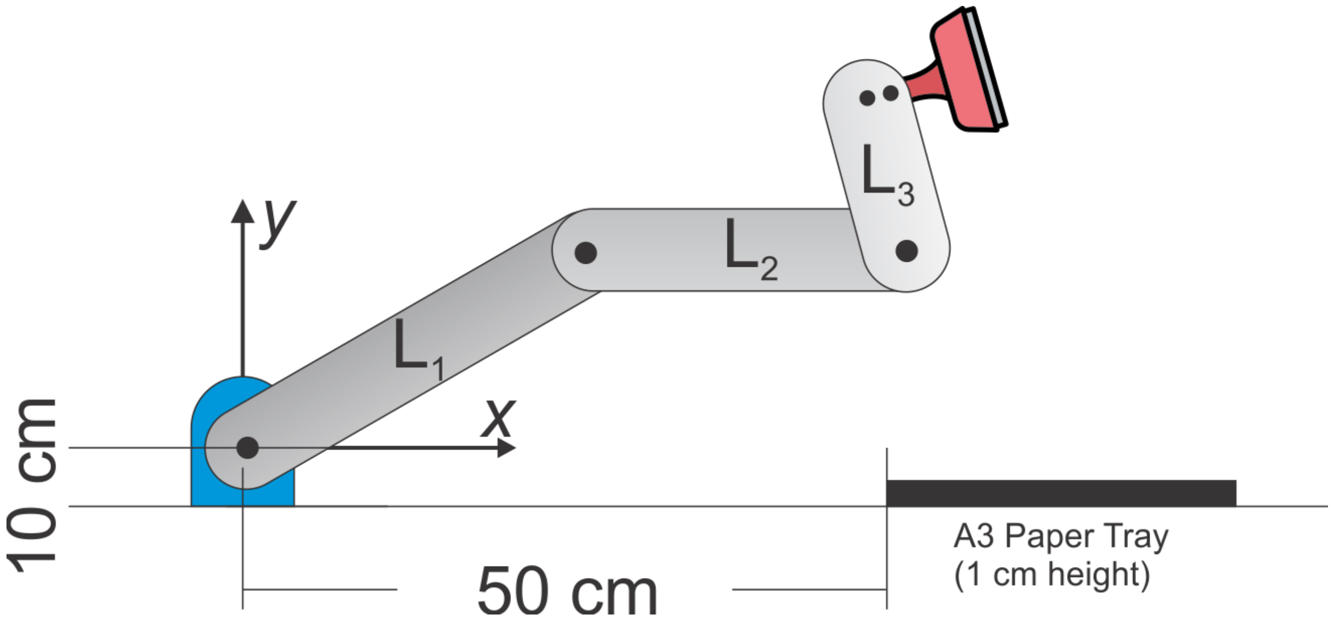
b. If yes, determine the Roll, Pitch, and Yaw that define each matrix. Please state your assumptions. Do you believe their values?

2. The Odd Chance It Could Work [14%]

Review the even and odd nature of sine/cosine functions. Using this, explain how many positive and negative terms you should expect might change in a rotation matrix if a Z-Y-X moving Euler Angle triad if angles were given clockwise positive (or left-handed) instead of counterclockwise positive (i.e. as a negative angle).

3. **A Modest Stamp Duty** **[35%]**

A small humanoid robot is being programmed to duly stamp to a form (for which a duty is collected). The first objective is to place the stamp on a form placed in the 42 cm (A3) wide, 1 cm thick paper tray (solid back outline) in the figure below. Assume that the arm is composed of 3 revolute joints and is constrained to move in the plane of the page. The arm consists of 3 links with joint-to-joint dimensions: L_1=0.5 m, L_2=0.3 m, L_3=0.2 m (to the stamp's mounting). The stamp is riveted orthogonally to the last link (L_3) and is placed such that it extends 10 cm from the rivet location. Assume initially that the stamp must be parallel (i.e., of the same angle) as the form/table.



A. **Getting the Solution in the Right Form**
Please calculate analytically the **forward and inverse kinematics** for this system.

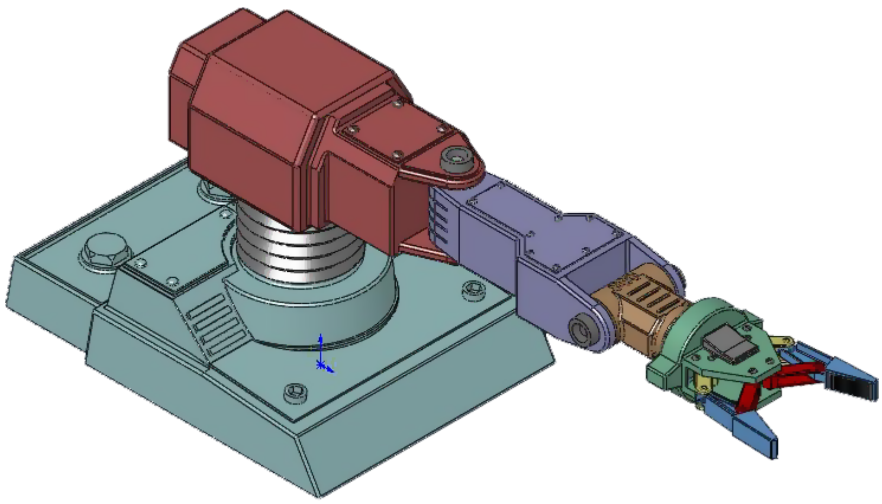
B. **Shuffling Paperwork**
Using the above (Part A), determine the robot's workspace and valid locations for placing the form such that it gets correctly stamped. **Now**, imagine the tray will be moved. What are both the **minimum and maximum** ranges of the paper-tray location (i.e., the "50 cm" value in the figure above) such that robot can place a stamp for the entirety of the tray?

C. **Inking a Solution**
Please calculate a trajectory function to take the arm from an initial pose to a final pose. Assuming initial arm angles of $\theta_1=0^o$, $\theta_2=15^o$, $\theta_3=120^o$, then plot a valid trajectory for a form at the center of the tray placed at the middle of range determined in the previous step (Part B).

D. **A Riveting Conclusion**
Please modify the trajectory function from above (Part C) so to allow for placement stamp if the second (lower) rivet holding the stamp in place breaks so that the stamp itself may rotate freely about the first (upper) rivet. **How is this solution changed** if the stamp is modified so that instead of having to be parallel to the form, it can be within ± 15 degrees of the paper for a correct mark to be placed?

4. **Back to the Future -- Forward & Differential Kinematics** **[35%]**

Consider the Armatron robot shown in the figure below

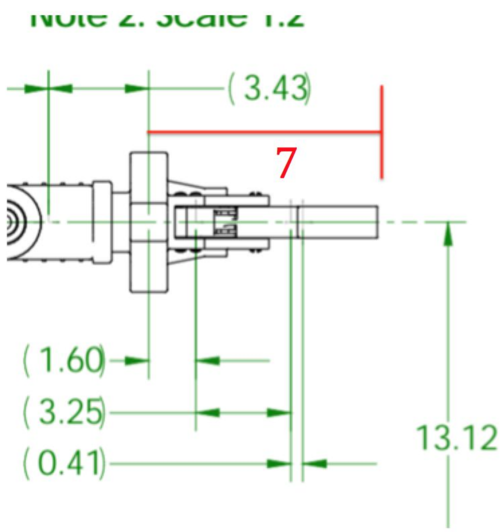


(See links for enlarged views of the arm [**png**], solid model [**stl**], and **its dimensions** [**pdf**])

a. Please form a kinematic diagram of the arm, including link lengths.

b. Adopting the "Standard" Denavit-Harenberg (D-H) convention, what are the D-H parameters/table for the robot? (i.e., for each link (i), the parameters $\boldsymbol{a_i}$, $\boldsymbol{\alpha_i}$, $\boldsymbol{d_i}$, $\boldsymbol{\theta_i}$)

c. For this arm, please derive the forward kinematics for the centre (tip when closed) of the end-effector (the blue part in the above isometric CAD view) given the length of the gripper is 7 cm as shown below.

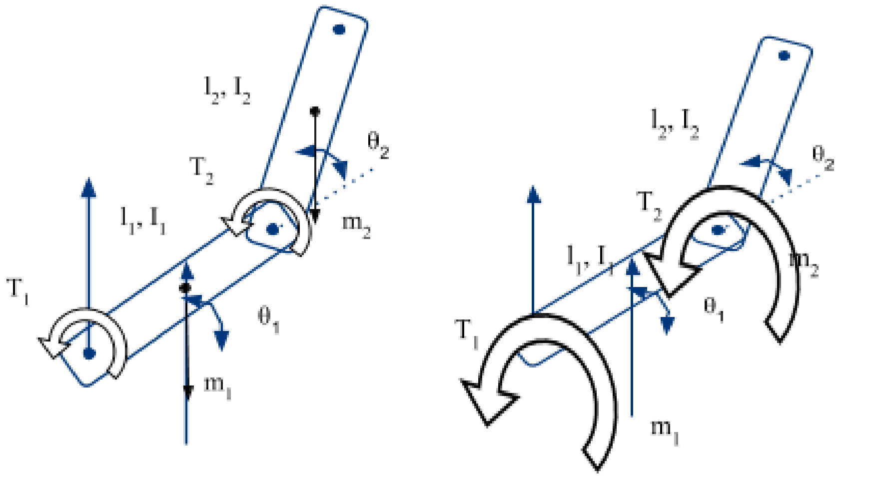


d. What is the angular velocity of the tip of the end-effector in terms of the joint rates? (Please express this in a fixed frame at the origin).

5. **EXTRA CREDIT — Gravity!** **[Up to +10% EXTRA]**

Well beyond popular culture, gravity is a subject of deep space and thought. It is also a critically important subject in robotics, such as for robot arm dynamics. Consider the two link manipulator below in both a vertical and horizontal configuration (relative to gravity).

(a) vertical configuration (gravity ↓)   (b) horizontal configuration (gravity into page ⓧ)



$l_1 = 0.75$ m	$l_2 = 0.5$m
$m_1 = 4$ kg	$m_2 = 3$ kg
$I_1 = 0.4$ kgm^2	$I_2 = 0.2$ kgm^2
$\theta_1 =$ joint angle 1	$\theta_2 =$ joint angle 2
$\boldsymbol{\tau}_1$ = torque at joint 1	$\boldsymbol{\tau}_2$ = torque at joint 2

a. Derive the equations of motion that describe the system dynamics of the two configurations.

b. For a given torque-limited motor ($\boldsymbol{\tau}^*_{max}$), what is the maximum load (weight) the robot can move in each (i.e., vertical and horizontal) configuration?

c. Based on the results above, please describe the effect of gravity on robot arm dynamics.

Due Date

The problem set must be completed **individually** by the end of Week 5 (23:59 AEST Friday**, August 24, 2018**). Submission is via Platypus.

Early submission is **highly** encouraged.