## Linear Observers & LQR + Course Review

**METR4202: Advanced Control & Robotics**

Dr Surya Singh -- Lecture # 13  

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### Schedule

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<td><strong>Linear Observers &amp; LQR + Course Review</strong></td>
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Announcements: Lab 3 Extended

- **Lab 3:**
  - Due Nov 3 or Nov 12
  - Time Signup online
  - Rubric online

- **Individual Assignment**
  - Online too!
  - Just attempt 100 points worth
    - That’s 50% -- the entire paper is worth 200 points!
    - **No extra credit** for trying additional problems >100 points.

- **Cool Robotics Share Site**
  Twitter: [#metr4202](https://twitter.com/search?q=#metr4202)

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Shaping of Dynamic Responses
Let’s Generalize This

• Shaping the Dynamic Response
  – A method of designing a control system for a process in which all the state variables are accessible for measurement—the method known as pole-placement

• Theory:
  – We will find that in a controllable system, with all the state variables accessible for measurement, it is possible to place the closed-loop poles anywhere we wish in the complex $s$ plane!

• Practice:
  – Unfortunately, however, what can be attained in principle may not be attainable in practice. Speeding the response of a sluggish system requires the use of large control signals which the actuator (or power supply) may not be capable of delivering. And, control system gains are very sensitive to the location of the open-loop poles.

Regulator Design

• Here the problem is to determine the gain matrix $G$ in a linear feedback law
  \[ u = -Gx - G_0x_0 \]
  – Where: $x_0$ is the vector of exogenous variables. The reason it is necessary to separate the exogenous variables from the process state $x$, rather than deal directly with the metastate $x = \begin{bmatrix} x \\ x_0 \end{bmatrix}$ is that we must assume that the underlying process is controllable.
  
  • Since the exogenous variables are not true state variables, but additional inputs that cannot be affected by the control action, they cannot be included in the state vector when using a design method that requires controllability.
  
  • HOWEVER, they can be used in a process for Observability! ∴ when we are doing this as part of the sensing/mapping process!!
• The assumption that all the state variables are accessible to measurement in the regulator means that the gain matrix $G$ in is permitted to be any function of the state $x$ that the design method requires

$$y = Cx$$
$$u = -G_y y$$
$$u = -G' \dot{x}$$

– Where: $\dot{x}$ is the state of an appropriate dynamic system known as an "observer."

SISO Regulator Design

• Design of a gain matrix

$$G = g' = [g_1, g_2, \ldots , g_k]$$

for the single-input, single-output system

$$x = Ax + Bu$$

where

$$B = b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix}$$

With the control law $u = -Gx = -g'x$ (6.7) becomes

$$\dot{x} = (A - bg')x$$

• Our objective is to find the matrix $G = g'$ which places the poles of the closed-loop dynamics matrix $A_c = A - bg'$ at the locations desired.
SISO Regulator Design [2]

- One way of determining the gains would be to set up the characteristic polynomial for $Ac$:

$$|sI - A_c| = |sI - A + bg'| = s^k + \tilde{a}_1s^{k-1} + \cdots + \tilde{a}_k$$

- The coefficients $a_1, a_2, \ldots, a_k$ of the powers of $s$ in the characteristic polynomial will be functions of the $k$ unknown gains. Equating these functions to the numerical values desired for $a_1, a_2, \ldots, a_k$ will result in $k$ simultaneous equations the solution of which will yield the desired gains $g_1, \ldots, g_k$.

SISO Regulator Design [3]

If the original system is in the companion form given in (3.90), the task is particularly easy, because

$$A = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{k-1} & -a_k \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \quad (6.11)$$

$$by' = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} [g_1, g_2, \ldots, g_k] = \begin{bmatrix} g_1 & g_2 & \cdots & g_k \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

Hence

$$A_c = A - bg' = \begin{bmatrix} -a_1 - g_1 & -a_2 - g_2 & \cdots & -a_{k-1} - g_{k-1} & -a_k - g_k \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

The gains $g_1, \ldots, g_k$ are simply added to the coefficients of the open-loop $A$ matrix to give the closed-loop matrix $A_c$. This is also evident from the block-diagram representation of the closed-loop system as shown in Fig. 6.1.
SISO Regulator Design [4]

- But how to get this in companion form?

\[ \ddot{x} = Tx \]  

(6.14)

Then, as shown in Chap. 3,

\[ \dot{x} = \tilde{A}x + \tilde{b}u \]  

(6.15)

where

\[ \tilde{A} = TAT^{-1} \quad \text{and} \quad \tilde{b} = Tb \]

For the transformed system the gain matrix is

\[ \tilde{g} = \tilde{a} - \tilde{a} = \tilde{a} - a \]  

(6.16)

since \( \tilde{a} = a \) (the characteristic equation being invariant under a change of state variables). The desired control law in the original system is

\[ u = -g'x = -g'T^{-1}\ddot{x} = -\tilde{g}'\ddot{x} \]  

(6.17)

From (6.17) we see that

\[ \tilde{g}' = g'T^{-1} \]

Thus the gain in the original system is

\[ g = T'\tilde{g} = T(\tilde{a} - a) \]  

(6.18)
SISO Regulator Design [5]

In words, the desired gain matrix for a general system is the difference between the coefficient vectors of the desired and actual characteristic equation, premultiplied by the inverse of the transpose of the matrix $T$ that transforms the general system into the companion form of (3.90), the $A$ matrix of which has the form (6.11).

The desired matrix $T$ is obtained as the product of two matrices $U$ and $V$:

$$ T = VU $$  

(6.19)

The first of these matrices transforms the original system into an intermediate system

$$ \dot{x} = \tilde{A}\dot{x} $$  

(6.20)

in the second companion form (3.107) and the second transformation $U$ transforms the intermediate system into the first companion form.

Consider the intermediate system

$$ \dot{x} = \tilde{A}\dot{x} + \tilde{b}u $$  

(6.21)

with $\tilde{A}$ and $\tilde{b}$ in the form of (3.107). Then we must have

$$ \tilde{A} = UAU^{-1} \quad \text{and} \quad \tilde{b} = Ub $$  

(6.22)

SISO Regulator Design [6]

The desired matrix $U$ is precisely the inverse of the controllability test matrix $Q$ of Sec. 5.4. To prove this fact, we must show that

$$ U^{-1} \tilde{A} = AU^{-1} $$  

(6.23)

or

$$ QA \tilde{A} = AQ $$  

(6.24)

Now, for a single-input system

$$ Q = [b, Ab, \ldots, A^{k-1}b] $$

Thus, with $\tilde{A}$ given by (3.107), the left-hand side of (6.23) is

$$ Q\tilde{A} = \begin{bmatrix}
0 & 0 & \cdots & -a_k \\
1 & 0 & \cdots & -a_{k-1} \\
& 1 & \cdots & -a_{k-2} \\
& & \cdots & \cdots \\
& & & 0 & \cdots & -a_1
\end{bmatrix} $$

$$ = [Ab, A^2b, \ldots, A^{k-1}b, -a_kb, -a_{k-1}Ab, \cdots, -a_1A^{k-1}b] $$  

(6.25)

The last term in (6.25) is

$$ (-a_1I - a_{k-1}A - \cdots - a_1A^{k-1})b $$  

(6.26)
SISO Regulator Design [7]

Now, by the Cayley-Hamilton theorem, (see Appendix):

\[ A^k = -a_1 A^{k-1} - a_2 A^{k-2} - \ldots - a_d I \]

so (6.26) is \( A^k b \). Thus the left-hand side of (6.24) as given by (6.25) is

\[ Q \tilde{A} = [A b, A^2 b, \ldots, A^d b] = A[b, A b, \ldots, A^{d-1} b] = QA \]

which is the desired result.

If the system is not controllable, then \( Q^{-1} \) does not exist and there is no general method of transforming the original system into the intermediate system (6.21); in fact it is not possible to place the closed-loop poles anywhere one desires. Thus, controllability is an essential requirement of system design by pole placement. If the system is *stabilizable* (i.e., the uncontrollable part is asymptotically stable, as discussed in Chap. 5) a stable closed-loop system can be achieved by placing the poles of the controllable subsystem where one wishes and accepting the pole locations of the uncontrollable subsystem. In order to apply the formula of this section, it is necessary to first separate the uncontrollable subsystem from the controllable subsystem.

The control matrix \( \tilde{b} \) of the intermediate system is given by

\[ \tilde{b} = Ub \quad (6.27) \]

We now show that

\[ \tilde{b} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (6.28) \]

SISO Regulator Design [8]

Multiply (6.28) by \( Q \) to obtain

\[ Q \tilde{b} = [A b, A^2 b, \ldots, A^d b] = b \]

which is the same as (6.27), since \( Q^{-1} = U \).

The final step is to find the matrix \( V \) that transforms the intermediate system (6.21) into the final system (6.15). We must have

\[ \tilde{x} = V \tilde{x} \quad (6.29) \]

For the transformation (6.28) to hold, we must have

\[ \tilde{A} = V \tilde{A} V^{-1} \]

or

\[ V^{-1} \tilde{A} = \tilde{A} V^{-1} \quad (6.30) \]
SISO Regulator Design [9]

The matrix $V^{-1}$ that satisfies (6.30) is the transpose of the upper left-hand $k$-by-$k$ submatrix of the (triangular Toeplitz) matrix appearing in (3.103)

$$V^{-1} = \begin{bmatrix} 1 & a_1 & \cdots & a_{k-1} \\ 0 & 1 & \cdots & a_{k-2} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} = W \tag{6.31}$$

To prove this, we note that the left-hand side of (6.30) is

$$V^{-1} \tilde{A} = \begin{bmatrix} 1 & a_1 & \cdots & a_{k-1} & -a_1 & -a_2 & \cdots & -a_k \\ 0 & 1 & \cdots & a_{k-2} & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_k \\ 1 & a_1 & \cdots & a_{k-2} & 0 \\ 0 & 1 & \cdots & a_{k-3} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \end{bmatrix} \tag{6.32}$$

(Note that the zeros in the first row of $V^{-1} \tilde{A}$ are the result of the difference of...)

SISO Regulator Design [10]

Two terms $a_i - a_1$, $a_2 - a_1$, etc.) and the right-hand side of (6.30) is

$$\tilde{A} V^{-1} = \begin{bmatrix} 0 & 0 & \cdots & -a_1 \\ 1 & a_1 & \cdots & a_{k-1} \\ 0 & 1 & \cdots & a_{k-2} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -a_k \end{bmatrix} \begin{bmatrix} 1 & a_1 & \cdots & a_{k-1} \\ 1 & 0 & \cdots & -a_1 \\ 0 & 1 & \cdots & a_{k-2} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -a_k \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_k \\ 1 & a_1 & \cdots & a_{k-2} & 0 \\ 0 & 1 & \cdots & a_{k-3} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \end{bmatrix}$$

which is the same as (6.32). Thus (6.30) is proved.

We also need $\delta - V \delta$

We will show that $\delta = \delta$

Consider $\delta = V^{-1} \delta$

with

$$b = V^{-1} \delta = \begin{bmatrix} 1 & a_1 & \cdots & a_{k-1} \\ 0 & 1 & \cdots & a_{k-2} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$
SISO Regulator Design [11]

Thus $\tilde{b}$ and $\tilde{a}$ are the same.

The result of this calculation is that the transformation matrix $T$ whose transpose is needed in (6.18) is the inverse of the product of the controllability test matrix and the triangular matrix (6.31).

The above results may be summarized as follows. The desired gain matrix $g$, by (6.18) and (6.19), is given by

$$g = (VU)Y(\tilde{a} - a)$$

(6.33)

where

$$V = W^{-1} \quad \text{and} \quad U = Q^{-1}$$

Thus

$$VU = W^{-1}Q^{-1} = (QW)^{-1}$$

LQR
Deterministic Linear Quadratic Regulation

Figure 20.1 shows the feedback configuration for the linear quadratic regulation (LQR) problem. The process is assumed to be a continuous-time LTI system of the form

\[
\begin{align*}
\dot{x} &= Ax + Bu, & x \in \mathbb{R}^n, \\ y &= Cx, & y \in \mathbb{R}^m, \\ z &= Gx + Hu, & z \in \mathbb{R}^l,
\end{align*}
\]

and has two distinct outputs.

1. The measured output \( y(t) \) corresponds to the signal(s) that can be measured.

\[
\begin{array}{c}
\text{controller} \\
\downarrow \\
\text{process} \\
\downarrow \\
y(t) \in \mathbb{R}^m
\end{array}
\]

2. The controlled output \( z(t) \) corresponds to the signal(s) that one would like to make as small as possible in the shortest possible time. Sometimes \( z(t) = y(t) \), which means that our control objective is simply to make the measured output very small. At other times one may have

\[
z(t) = \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix},
\]

which means that we want to make both the measured output \( y(t) \) and its derivative \( \dot{y}(t) \) very small. Many other options are possible.
Optimal Regulation

The LQR problem is defined as follows. Find the control input $u(t)$, $t \in [0, \infty)$ that makes the following criterion as small as possible:

$$J_{LQR} := \int_0^\infty \|z(t)\|^2 + \rho \|u(t)\|^2 dt,$$  \hspace{1cm} (20.1)

where $\rho$ is a positive constant. The term

$$\int_0^\infty \|z(t)\|^2 dt$$

corresponds to the energy of the controlled output, and the term

$$\int_0^\infty \|u(t)\|^2 dt$$

corresponds to the energy of the control signal. In LQR one seeks a controller that minimizes both energies. However, decreasing the energy of the controlled output will require a large control signal, and a small control signal will lead to large controlled outputs. The role of the constant $\rho$ is to establish a trade-off between these conflicting goals.

Optimal Regulation

1. When we chose $\rho$ very large, the most effective way to decrease $J_{LQR}$ is to employ a small control input, at the expense of a large controlled output.

2. When we chose $\rho$ very small, the most effective way to decrease $J_{LQR}$ is to obtain a very small controlled output, even if this is achieved at the expense of employing a large control input.

Often the optimal LQR problem is defined more generally and consists of finding the control input that minimizes

$$J_{LQR} := \int_0^\infty z(t)' \tilde{Q} z(t) + \rho u(t)' \tilde{R} u(t) dt,$$ \hspace{1cm} (20.2)
Optimal Regulation

where \( \tilde{Q} \in \mathbb{R}^{l \times l} \) and \( \tilde{R} \in \mathbb{R}^{m \times m} \) are symmetric positive-definite matrices and \( \rho \) is a positive constant.

We shall consider the most general form for a quadratic criterion, which is

\[
J_{\text{LQR}} := \int_0^\infty x(t)' \tilde{Q} x(t) + u(t)' \tilde{R} u(t) + 2x(t)' \tilde{N} u(t) \, dt. \tag{J-LQR}
\]

Since \( z = Gx + Hu \), the criterion in (20.1) is a special form of the criterion (J-LQR) with

\[
Q = G'G, \quad R = H'H + \rho I, \quad N = G'H
\]

and (20.2) is a special form of the criterion (J-LQR) with

\[
Q = G' \tilde{Q} G, \quad R = H' \tilde{Q} H + \rho \tilde{R}, \quad N = G' \tilde{Q} H.
\]

Optimal State Feedback

It turns out that the LQR criterion

\[
J_{\text{LQR}} := \int_0^\infty x(t)' Q x(t) + u(t)' R u(t) + 2x(t)' N u(t) \, dt \tag{I-LQR}
\]

can be expressed as in (20.3) for an appropriate choice of feedback invariant. In fact, the feedback invariant in Proposition 20.1 will work, provided that we choose the matrix \( P \) appropriately. To check that this is so, we add and subtract this feedback invariant to the LQR criterion and conclude that

\[
J_{\text{I-LQR}} := \int_0^\infty x(t)' Q x(t) + u(t)' R u(t) + 2x(t)' N u(t) \, dt
\]

\[
= H(x(\cdot); u(\cdot)) + \int_0^\infty x'(A'P + PA + Q)x + u'Ru + 2u'(B'P + N')x \, dt.
\]
Optimal State Feedback

By completing the square, we can group the quadratic term in $u$ with the cross-term in $u$ times $x$:

$$
(u' + x'K)R(u + Kx) = u'Ru + x'(PB + N)R^{-1}(B'P + N')x + 2u'(B'P + N)x,
$$

where

$$
K := R^{-1}(B'P + N'),
$$

from which we conclude that

$$
J_{LQR} = H(x(); u(); t) + \int_{0}^{\infty} x'(A'P + PA + Q - (PB + N)R^{-1}(B'P + N'))x + (u' + x'K')R(u + Kx) dt.
$$

Optimal State Feedback

If we are able to select the matrix $P$ so that

$$
A'P + PA + Q - (PB + N)R^{-1}(B'P + N') = 0, \quad (20.5)
$$

we obtain precisely an expression such as (20.3) with

$$
\Lambda(x, u) := (u' + x'K)R(u + Kx),
$$

which has a minimum equal to zero for

$$
u = -Kx, \quad K := R^{-1}(B'P + N'),
$$

leading to the closed-loop system

$$
\dot{x} = (A - BR^{-1}(B'P + N'))x.
$$

The following has been proved.

**Theorem 20.1.** Assume that there exists a symmetric solution $P$ to the algebraic Riccati equation (20.5) for which $A - BR^{-1}(B'P + N')$ is a stability matrix. Then the feedback law

$$
u(t) := -Kx(t), \quad \forall t \geq 0, \quad K := R^{-1}(B'P + N'),
$$

minimises the LQR criterion (J-LQR) and leads to

$$
J_{LQR} := \int_{0}^{\infty} x'Qx + u'Ru + 2x'Nu dt = x'(0)Px(0).
$$

\[\square\]
LQR In MATLAB

MATLAB® Hint 42 (lqr). The command \([K, P, E] = lqr(A, B, Q, R, N)\) solves the algebraic Riccati equation

\[
A'P + PA + Q - (PB + N)R^{-1}(B'P + N') = 0
\]

and computes the (negative feedback) optimal state feedback matrix gain

\[
K = R^{-1}(B'P + N')
\]

that minimizes the LQR criteria

\[
J := \int_0^\infty x'Qx + u'R u + 2x'Nu\,dt
\]

for the continuous-time process

\[
\dot{x} = Ax + Bu.
\]

This command also returns the poles \(E\) of the closed-loop system

\[
\dot{x} = (A - BK)x.
\]
“Bang-Bang Control!”

Perhaps Not… Certainly Non-Linear!
Gryphon: Mine Scanning Robot

Landmines: Smart for one, dumb for all...
Generalized Mine & Placement

(Antipersonnel) Landmines are Challenging

- Variable & Changing
- Terrain Diversity
- Counter-thwart mechanisms
- False Positive Rate >100:1
Ex: PMN-2 [1]

Ex: PMN-2 [2]
Ex: PMN-2 [3]: Mechanically Intricate

- Little metal
  → “High-sensitivity” detectors / instruments

- **Highly Variable**
  (Example: PMN-2):
  - 3-stage detonation
  - Anti-thwart
  - All mechanical
  - Poor construction detectors / instruments

  → Focus on manipulating sensor instead of complex sensing ???,
Clearance & Breeching

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<th>Humanitarian</th>
<th>Military</th>
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<tr>
<td>Detection</td>
<td>&gt;98%</td>
<td>50-60%</td>
</tr>
<tr>
<td>Rate (m²/day/person)</td>
<td>~ 200</td>
<td>~ 10,000</td>
</tr>
<tr>
<td>Conditions</td>
<td>Fair weather/daytime</td>
<td>All weather/24/7</td>
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<tr>
<td>Standards</td>
<td>Int. Mine Action Std.</td>
<td>Army Field Manuals</td>
</tr>
<tr>
<td>Funding (source)</td>
<td>Gov’t, NGOs</td>
<td>Military</td>
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- Breeching: Line
- Demining: Area
  → International Mine Action Standards (IMAS)

Humanitarian Demining Process
Humanitarian Demining Process

1. Level 1: Specification
   - Rough minefield location

2. Level 2: Clearing
   - Heavy machines
     (e.g., flails, grinders, rollers, ploughs, and sifters)
   - ~ 90% clearance

3. Level 3: Confirmation

Sensor Mobility Is Critical
Robust Control: Command Shaping for Vibration Reduction
Command Shaping

Original velocity profile

Input shaper

Command-shaped velocity profile

Command Shaping in Position Space

Position

Time

A₁ Response

A₂ Response

Total Response
Command Shaping:
Zero Vibration and Derivative

\[ K = e^{\left(\frac{-\zeta \pi}{\sqrt{1-\zeta^2}}\right)} \quad i = 1, 2 \]

\[
\begin{bmatrix}
A_i \\
t_i
\end{bmatrix} = \begin{bmatrix}
1 & 2K(1+K)^2 & K^2(1+K)^2 \\
(1+K)^2 & 0 & \frac{T_d}{2} \\
0 & \frac{T_d}{2} & T_d
\end{bmatrix}
\]

For Gryphon:

<table>
<thead>
<tr>
<th>[ \frac{\omega}{\zeta} ]</th>
<th>[ \frac{\omega}{\zeta} ]</th>
<th>( \frac{\rho - \rho_0}{\rho - \rho_1} )</th>
<th>( \omega )</th>
<th>( \zeta )</th>
<th>At ( \rho_0=1.5 \text{ [m]} )</th>
<th>At ( \rho_1=3.0 \text{ [m]} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>( \rho_0 )</td>
<td>( \rho_1 )</td>
<td>( \rho )</td>
<td>( \rho_0 )</td>
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Part of a Robotic Solution…

- Metal detector
- Optional ground-penetrating radar
- Stereo vision camera
- Network camera
- Counter-weight
- Cleared area
- Minefield
- All terrain vehicle
- Minefield detector
Gryphon Schematic

Gryphon: Comparison to other tracked robots

Control Robustness ("Autonomy")

Mechanical Robustness
Multiple Inaccuracies

- Sensing:
- ATV Suspension:

Operational Overview

- Terrain scanning
- Heightmap
- Detecting

Terrain
- Environment Occupancy
- Grid

Sensing (Stereo Vision)
- Calibration Model (Offline Data)
- Noisy terrain data

Planar conditional filtering & map generation (Online Data)
- Terrain model relative to robot base with offset
- Path generation & collection correction
- Nominal Path

Command Shaping
- Final trajectory (with reduced vibration)
Terrain Modeling & Following Overview

• I. Terrain Mapping
• II. Terrain Model

• III. Path Generation
• IV. Scanning

V. Evaluation & Marking

Terrain Mapping

• Stereo depth maps (Pont Gray Bumblebee)
• Kinematic calibration corrections

Ex: Grassy area with hill or bump
Terrain Geometry Model: Heightmap Expansion

As Surface Normals:

Raw Map → Raw Model → Expanded and Offset → Filtered Model

Terrain Geometry Model: Conditional Planar Filter

- Planarity: Found from plane eq. residuals for a surface patch
- Filter type and strength varied based on this
- Goal: Reduce noise without feature degradation
Terrain Map $\rightarrow$ Model: Conditional Planar Filter

- Compute Normals
- Apply filter(s)

Map $\rightarrow$ Model (II): Height Map Expansion

- Envelope expansion:
  - $F_{env} = F_{terr} + \text{scanning gap} \ldots$
  - Performed along the normals, more than vertical axis addition:
Map → Model (III): Height Map Expansion

Calibration Model

- Height (z) Calibration:

- Plane (x-y) Calibration
Effect of Overall Calibration Matrix

Scanning speed: 100 mm/s
Scanning gap: 100 mm

Path Generation

- x-y: Scanning Scheme
- Joint-space/Work-space?
- Reduce excess work …
- z: Terrain Sampling (z)
- Sample corresponding point based on the local patch & normals

\[ z_{path} = f_{env}(x_{path}, y_{path}) \]
Path Generation (II)

- Orientation: Advanced Terrain Following

Contour Following
Terrain Modeling:
Find a good model to characterize

Experiments: Scanning Over Obstacle
Scanning on ~ Level Terrain - Measurements

Scanning speed: 100 mm/s
Scanning gap: 100 mm

Scanning on Rough Terrain - Measurements

Scanning speed: 100 mm/s
Scanning gap: 100 mm
Command Shaping Tests: Step-Response

- Reduced Joint Encoder Vibration
- Reduced Tip Acceleration

Joint 1 (ATV Yaw) Encoder:

Joint 3 (Arm Extend) Encoder:

High-Level Control Software
Detector Imaging

• Targets

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Extensive Field Tests

2005: Kagawa, Japan
2006: Benkovac, Croatia
2007: Siem Reap, Cambodia
Gryphon: Field Tests in Croatia & Cambodia

February 2006: Tests in Croatia

Metal detector imaging
Terrain & Estimation

- IF we know terrain → Triangulation
- IF we know depth → SNR gives terrain “characteristic”
- → Estimate both simultaneously (→ solution up to scale)

End on a “Bang, Bang”…
A Better (Controlled) “Bang Bang”

Where to from here? Natural Motion
UQ Robotics: Dynamic Systems in Motion

Diverse international research group

**Hanna Kurniawati**
(NUS/MIT)

**Paul Pounds**
(ANU/Yale)

**Surya Singh**
(Stanford/Syd)

Aerial Systems

Bio-inspired Systems

Mechanics of motion

SECaT Time! … Brought To You By the Number 5