The objective of this tutorial is to explore homogenous transformations. The MATLAB robotics toolbox developed by Peter Corke might be a useful aid.

Reading
Please read/review chapter 7 of Robotics, Vision and Control.

Review
Useful commands:
Transl, trotx, troty, trotz, rotx, roty, rotz, tr2eul, DHFactor

Familiarise yourself with the link class

Questions
1. For the robot shown in the following figure, find the table of DH parameters according to “Standard” DH conventions.
   (note: you are allowed to move the initial frame to fit convention(s))

![Diagram of a robot with labeled axes and joints]

Answers:

<table>
<thead>
<tr>
<th>Link</th>
<th>From Frame</th>
<th>To Frame</th>
<th>( \theta_i )</th>
<th>( d_i )</th>
<th>( a_i )</th>
<th>( \alpha_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>01*</td>
<td>4</td>
<td>1</td>
<td>-90°</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>02*</td>
<td>0</td>
<td>2</td>
<td>90°</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>03*</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that the position of the end effector (the gripper) may be viewed as a position vector \( \mathbf{p}_{\text{end effector}} \) in Frame 3.
a.) Determine the joint angles of the two-link planar arm.

The joint space of the robot is $(\theta_1, \theta_2)$.

The forward kinematics may be solved directly using the vector-loop method or somewhat more mechanically using the DH convention (see slides 24 and 42 of Lecture 3). This gives:

$$(p_x, p_y) = (a_1c\theta_1 + a_2c\theta_2, \quad a_1s\theta_1 + a_2s\theta_2)$$

The inverse kinematics involves solving the above simultaneous equation for $\theta_1$ and $\theta_2$.

A geometric way of solving this is to observe that the distance from $\{0\}$ to $\{2\}$ is independent of $\theta_1$. Thus, sum of squares gives:

$$p_x^2 + p_y^2 = a_1^2 + a_2^2 + 2a_1a_2c\theta_2$$

$$\theta_2 = \arccos\left(\frac{p_x^2 + p_y^2 - a_1^2 - a_2^2}{2a_1a_2}\right)$$

If $\theta^*$ is an answer to the above, the, in general, $-\theta^*$ will also be an answer. This corresponds to the “elbow up” and “elbow down” configurations.

Substituting this back into the kinematic equations gives:

$$p_x = (a_1 + a_2c\theta_2)c\theta_1 + (a_2s\theta_2)s\theta_1, \quad p_y = (a_2s\theta_2)c\theta_1 + (a_1 + a_2c\theta_2)s\theta_1$$

$$c\theta_1 = \frac{p_x(a_1 + a_2c\theta_2) + p_y(a_2s\theta_2)}{a_1^2 + a_2^2 + 2a_1a_2c\theta_2}$$

$$s\theta_1 = \frac{-p_x(a_2s\theta_2) + p_y(a_1 + a_2c\theta_2)}{a_1^2 + a_2^2 + 2a_1a_2c\theta_2}$$

$$\theta_1 = \text{Atan2}(s\theta_1, c\theta_1)$$
If $a_1 = 2$ and $a_2 = 3$ what are the joint angles corresponding to an end effector position of $(x,y)=(1, 1)$.

$$
\theta_1 = 167.028^\circ, \theta_2 = -156.44^\circ \text{ (Elbow down)} \\
\text{Or } \theta_1 = -77.028^\circ, \theta_2 = 156.44^\circ \text{ (Elbow up)}
$$

To verify using the Robotics Toolbox:

```matlab
L(1) = Link([ 0 0 2 0], 'standard')
L(2) = Link([ 0 0 3 0], 'standard')
twolink = SerialLink(L, 'name', 'two link')
T=rpy2tr(0,0,0); T(1:2, 4)=[1 1]
Qsol=twolink.ikine(T, zeros(1,2), [1 1 0 0 0 0])
```

$Qsol = \begin{bmatrix} 2.9152 \\ -2.7305 \end{bmatrix}$