Sensor Fusion and Filtering

or

“Making sensors make sense”

Paul Pounds

19 March 2013
University of Queensland
But first…

Some house keeping
# Calendar at a glance

<table>
<thead>
<tr>
<th>Week</th>
<th>Dates</th>
<th>Lecture</th>
<th>Reviews</th>
<th>Demos</th>
<th>Assessment submissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25/2 – 1/3</td>
<td>Introduction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4/3 – 8/3</td>
<td>Principles of Mechatronic Systems design</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>11/3 – 15/3</td>
<td>Principles of Sailing</td>
<td>Design brief</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>18/3 – 22/3</td>
<td>Sensor Fusion and Filtering</td>
<td><strong>Progress review 1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>25/3 -29/3</td>
<td>???</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Break</td>
<td>1/4 – 5/4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>8/4 – 12/4</td>
<td>By request</td>
<td>Progress seminar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>15/4 – 19/4</td>
<td>By request</td>
<td></td>
<td>25% demo</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>22/4 – 26/4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>29/4 – 3/5</td>
<td></td>
<td>Progress review</td>
<td>50% demo</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>6/5 – 10/5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>13/5 – 17/5</td>
<td></td>
<td></td>
<td>75% demo</td>
<td>Preliminary report</td>
</tr>
<tr>
<td>12</td>
<td>20/5 – 24/5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>27/5 – 31/5</td>
<td>Closing lecture</td>
<td>Final testing</td>
<td>Final report and addendum</td>
<td></td>
</tr>
</tbody>
</table>
FAQ Roundup

- Do rudders count towards the hull dimensions?
  - No – they can extend beyond the 150 mm x 75 mm bounding box (but then will be invalid for scoring).
Next week’s lecture

• Nobody nominated anything.
  – Seriously? Why would you not do that?

• Ok, ok – don’t panic. We can fix this.

• I propose to instead run a best-practices soldering tutorial during the lecture time
  – Because your soldering is terrible (probably).
Progress Review

• Show you have been doing stuff!
  – You will have 3-5 minutes to demonstrate your contribution to the team

• Bring evidence!
  – Sketches, notes, prototypes, analysis, work breakdowns, etc. are all good.

• Pass/fail assessment
  – It should be difficult to fail this if you have actually done something useful
Progress Review sessions

Group times:

**Wed 20**
- Group ?  9:00-9:30
- Group 15  9:30-10:00
- Group 8   10:00-10:30
- Group 14  10:30-11:00
- Group 4   14:00-14:30 A
- Group 2   14:00-14:30 B
- Group 1   14:30-15:00 A
- Group 7   14:30-15:00 B

**Thursday 21**
- Group 13  9:00-9:30
- Group 11  9:30-10:00
- Group 10  10:00-10:30
- Group 5   10:30-11:00
- Group 3   13:00-13:30
- Group 6   13:30-14:00
- Group 9   14:00-14:30
- Group 12  14:30-15:00

Group B sessions are held in Axon 211, all other sessions are in GPS 310
Progress Review flow chart

- Are you at the meeting?
- Have you done anything useful?
- Not just the stuff other folks did?
- Have you got evidence for it?
- Is it a real contribution?
- Pass
Onwards to sensor fusion

To fusion and beyond!!
What is this sensing stuff?

- How systems find out about the world
  - And themselves

- Sensing is the measurement of some physical property of the environment
  - Physical property is analogous to the measurand, related to the state value of interest
  - Physical signal is typically transduced into an electrical signal (and often digitised)
Snuh?

• Sensors use a physical property to produce a signal related to the thing being measured.

Right.
An incomplete sensor taxonomy

<table>
<thead>
<tr>
<th>Domain</th>
<th>Modality</th>
<th>Example physics</th>
<th>Example sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intrinsic</td>
<td>‘Internal sense’</td>
<td>Mechanical</td>
<td>Gravity float</td>
</tr>
<tr>
<td>Nociception</td>
<td></td>
<td>Electrical</td>
<td>Current draw</td>
</tr>
<tr>
<td>Proprioception</td>
<td></td>
<td>Electromechanical</td>
<td>Strain gauge</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Optical</td>
<td>Rotary encoder</td>
</tr>
<tr>
<td></td>
<td>Equilibrioception</td>
<td>Optical</td>
<td>Ring laser gyro</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Electromechanical</td>
<td>MEMS gyro</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Microfluidic</td>
<td>Vestibular gyro</td>
</tr>
<tr>
<td>Boundary</td>
<td>Tactition</td>
<td>Electromechanical</td>
<td>Switch</td>
</tr>
<tr>
<td>Gustation</td>
<td></td>
<td></td>
<td>CH₄ detector</td>
</tr>
<tr>
<td>Olfaction</td>
<td></td>
<td>Electrochemical</td>
<td>Hyrgonometer</td>
</tr>
<tr>
<td>Audition</td>
<td></td>
<td>Electromechanical</td>
<td>Microphone</td>
</tr>
<tr>
<td>Extrinsic</td>
<td>Vision</td>
<td>Photoelectric</td>
<td>Camera</td>
</tr>
<tr>
<td>Lateration</td>
<td></td>
<td>Acoustic</td>
<td>Optic flow</td>
</tr>
<tr>
<td>Magnetoception</td>
<td></td>
<td>Photoelectric</td>
<td>Sonar</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Electromagnetic</td>
<td>3D scanner</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>GPS</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Radar</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Compass</td>
</tr>
</tbody>
</table>
More deeply

- Measurement is an attempt to find the true value of some real state parameter
  - Measurements and true states generally differ

- For practical, entropic, budgetary and philosophical reasons, no sensor is perfect.
  - Some are merely ‘adequate’.
Quick terminology

- **State**: numeric parameterisation of a thing
- **Signal**: time-varying state function that conveys information
- **True state**: real state being sought
- **Measurement**: signal from a sensor
- **Estimate**: inferred guess of the true state
Signals imperfect

• Measurement signals can be thought of as containing information about the true value

• Confounding effects obscure information
  – Entropic noise (eg. thermal noise)
  – Coupled noise/cross-talk
  – Bias
  – Nonlinearity
Postcards from the front

Rotor Speed Noise and Disturbance PSD

Tethered Indoor Attitude Stabilisation

Line Noise
Automation
Obstacle
Reference

Roll
Pitch

Angle (deg)

Time (s)

Frequency (Hz)

10^1
10^2

19 March 2013
Filtering

• Filters act on signals to remove confounding effects and extract information

• Many common examples:
  – Low pass filter: remove high frequency noise
  – High pass filter: remove low frequency bias
  – Common-mode filter: remove line coupling

Use multiple measurements of a signal in time to estimate the true state – temporal diversity
The intuitive idea

• Given time-history measurements of a state, what is its ‘most likely’ true value?
The intuitive idea

- The time-history of the measurement signal conveys information of the true value
The intuitive idea

- Examining temporal properties of the signal allow us to infer the true value

\[ x(t) \]

Signal bounds
The intuitive idea

- Examining temporal properties of the signal allow us to infer the true value
Depressing truth

A simple RC or ‘exponential’ 1st order filter will solve 80% of your practical noise elimination or smoothing problems*

• Quick, easy, and a snap to code/build!

*The other 20% requires an engineering degree or two
Sensor fusion

• Combination of multiple different sensors
  – Each sensor adds information (even poor ones), allowing for more accurate estimates
  – Sensors must measure the same states, or states relatable through a system model
    eg. compass and heading gyro to estimate bearing

Use multiple sensor modalities to estimate the true state – measurement diversity
The intuitive idea

- $n$ sensors can produce more accurate estimates together than possible individually.
The intuitive idea

- $n$ sensors can produce more accurate estimates together than possible individually

\[ P(x) \]

\[ x_1 \quad x_2 \quad \ldots x_n \]

True value $\rightarrow$ Simple average
The intuitive idea

- Sensor mean and variance encode more information than an instantaneous sample
The intuitive idea

- Can incorporate stochastic signal behaviour
Fundamental requirement

• Filtering/fusion tacitly assume we know something about the signal being sought:
  – Frequency band of true value/noise
  – Relative amplitude/power
  – Waveform shape or encoding

• Use knowledge to isolate known properties of the signal and suppress spurious effects
  (Naturally, you can also filter fused estimates)
Ergodic principle

• Over long enough time scales, a constant signal is equal to its mean
  – In ergodic dynamical systems of constant energy, all states are visited equally often

• Holds in the case of normal (Gaussian) distributions
  – This is why averaging works – static ‘window’
  – Not necessarily the case for ‘coloured’ noise
That’s great… but?

• How do we get that fused estimate?
• Several ways:
  – Weighted averaging
  – Least squares
  – Kalman filter
  – Particle filter
  – Complementary filter
  – Linear observer
  – Bayesian network
  – Dempster-Shafer

Key point: subsequent measurements cause estimates (and their variance) to converge
The Kalman filter

- Suppose we have a noisy measurement of our current state, with some estimate of variance
- If we know the system dynamics we can guess what the next state will be (with variance)
  - Compare where we think we should be to where our sensors tell us we are
  - Take a weighted average of the two, based on their covariance, as the new state
Eg. Pogo stick robot

• We know its approximate position and velocity
Eg. Pogo stick robot

• We know its approximate position and velocity
  – Using ballistics we can guess where it will land after each jump
Eg. Pogo stick robot

- We know its approximate position and velocity
  - Using ballistics we can guess where it will land after each jump... becoming less certain with time
Eg. Pogo stick robot

- We know its approximate position and velocity
  - Using ballistics we can guess where it will land after each jump... becoming less certain with time
Eg. Pogo stick robot

- We know its approximate position and velocity
  - Using ballistics we can guess where it will land after each jump… becoming less certain with time
  - After each jump, we get a new measurement and we can refine our estimate
Eg. Pogo stick robot

- We know its approximate position and velocity
  - Using ballistics we can guess where it will land after each jump... becoming less certain with time
  - After each jump, we get a new measurement and we can refine our estimate
Other approaches

• By now you should be completely sick of hearing about the Kalman filter
  – If not, go here:
    digi.physic.ut.ee/mw/images/d/d5/Poormankalman.pdf

• Let’s also look at the particle filter, the complementary filter and linear observer
  – Surya Singh also has a nice primer on estimators:
    robotics.itee.uq.edu.au/~metr3800/doc/ClassNotes_METR3800.pdf
The particle filter

• Similar concept, but uses discretely sampled estimates of the space of possible states
  – Simulate each step in time - track the particles
  – Find out how much each subsequent measurement agrees with each particle
  – Use the ‘best’ particle as the estimate
  – Occasionally resample around the most reliable particle
Eg. Kidnapped Robot Problem

- Robot moved to an unknown location
Eg. Kidnapped Robot Problem

- Robot moved to an unknown location
- Estimate we could be anywhere
Eg. Kidnapped Robot Problem

- Robot moved to an unknown location
- Estimate we could be anywhere – subsequent observations reduce likely candidate positions
Eg. Kidnapped Robot Problem

• Robot moved to an unknown location
• Estimate we could be anywhere – subsequent observations reduce likely candidate positions
Eg. Kidnapped Robot Problem

- Robot moved to an unknown location
- Estimate we could be anywhere – subsequent observations reduce likely candidate positions
Eg. Kidnapped Robot Problem

• Robot moved to an unknown location
• Estimate we could be anywhere – subsequent observations reduce likely candidate positions
Eg. Kidnapped Robot Problem

- Robot moved to an unknown location
- Estimate we could be anywhere – subsequent observations reduce likely candidate positions
Complementary filter

• Exploits heterogeneous sensor performance to overcome individual shortcomings

• Motivating application: MEMS IMU
  – Accelerometers: unbiased but very noisy
  – Gyroscopes: only kinda noisy but biased

Why not just use the accelerometers to correct the low-frequency bias of the gyros?
Complementary filter

- Exploit signal bandwidth properties:
  - Low pass filter accelerometer angle estimates
  - High pass filter gyros and integrate
  - Output is a weighted mix of estimates

Gratuitous name drop: ANU’s Prof. Rob Mahony, Paul’s PhD supervisor, wrote the complementary filter commonly used in UAV avionics stacks
Linear observers

Quick blast from the past semester: METR4202 observers

Note: if you haven’t done METR4202, don’t worry – this won’t be on the exam… and also there is no exam.
Linear observers

- Observers (aka “estimators”) are used to infer the hidden states of a system from measured outputs.

A controller is designed using estimates in lieu of full measurements.
Linear observers

• The state estimate can be treated like a control system itself
  – Dynamics to update the estimate:
    – Using an ‘error signal’, the difference between the real output measurement and the output estimate – the state estimate can be driven by a feedback term.
Linear observers

• Just like you might expect:

Choose observer feedback gain matrix $L$ to make converge to 0
Cross-contextualisation

- State-space observers are a sensor-fusion method that infers states from signals
  - But if observers are control functions…
  - And observers are filtering functions…

Profound realisation:

- Fundamentally, filtering is really control and control is really filtering!
  - Oh boy!
Some practical advice

• Some things engineers *never* try to build if they can buy, copy or otherwise avoid it:
  – Power supplies
  – Motor drivers
  – Analog amplifiers
  – Inertial Measurement Units
  – Sensor fusion and estimation algorithms

There are many good pre-canned S-F algorithms out there – try using them before writing your own!
Some practical advice

When combining sensors:

• Align measurements spatially and temporally
  – Calibrated kinematic transformation matrix?
  – Time-stamps, common interrupts lines?

• Use sensors to correct other sensors
  – Compensate motion of camera with IMU?
  – Augment dead-reckoning with optical flow?

• Reduce inter-sensor vibration/flex – rigidity!
Some practical advice

• Directly sense the state of interest, if possible
  – Avoid numerical integration or differentiation

• Hardware filters use fewer processor cycles

• Software filters take up less board (usually)

• Cheap sensor is cheap; better sensor is better
  – Fused cheap sensors might be almost as good
  – The easy solution is often a better sensors
Questions?
Tune-in next time for...

Your soldering is terrible

or

“How I learned to stop worrying and love flux”

Fun fact: One of the first practical applications of the Kalman filter was attitude estimation of the Apollo spacecraft.