1 DACs and ADCs

Both Digital-to-Analogue (DAC) and Analogue-to-Digital converters are exceptionally common devices that are used in more places than you would expect. So just how good are they? Let’s take a look.
1.1 ADCs

An ADC, put simply is a device that converts an analogue input into a digital quantity. There are a large number of ways that this can be done, although the most common types include "direct-conversion" ADC and successive approximation. Both of these come with pros and cons, typically in the form of accuracy vs speed.

A necessary trade-off for all ADCs is precision. Because there is a quantisation process used to produce a value for the received analogue signal, and this value is stored in the device, there is a limited amount of resolution - it is not possible to faithfully store the value of the captured signal in its entirety as this would require an infinite amount of precision and thus memory. Because generally speaking, we are not interested in reconstructing a signal including the noise, this is not inherently an issue. To illustrate the point, consider the following signal:

![A superposition of sinusoids of varying frequency](image)

Figure 1: A superposition of sinusoids of varying frequency

Now consider this being passed through a quantizer. A quantizer of any resolution only has a resolution of $2^N - 1$ where N is the number of bits available for storage in the device. As such, an 8-bit quantizer (such as a direct-conversion or "flash" ADC) has only 255 discrete voltage levels that it can represent, and it will pick the number closest (that is smaller than the actual input value). Can it represent the signal as it is passed to it?

If we quantize our previous signals using an 8-bit quantizer (7 if we look for signed values), there will be errors present. Zooming in to inspect our graph, we can see they are there:
As is pretty clearly visible, we have "steps", which are indicative of the different voltage levels that we are able to store (The y axis values are the binary number that correlates to the voltage - it has not been scaled by an order of magnitude as it may appear). The difference between our real value and our perceived value is known as quantisation error. In this case, the quantisation error is fairly low, but this could certainly be different for signals of higher frequencies.

Another problem to consider is the rate at which you can quantize your input signal. As you know, you can only sample so quickly and still reconstruct your signal - let’s try ramping it up:
Figure 3: At higher frequencies our quantized signal barely resembles our input.

Observe the almost entirely linear components between peaks and troughs. Note: this is after MATLAB did some unwanted interpolation - in reality this would be an almost-square wave - even worse.

1.2 DACs

DACs can be considered the opposite procedure to ADCs, and share a number of their issues - their precision and the time required to update the output value can easily distort a signal.

1.3 Reconstruction

With the function of these two in mind, it would be easy to conclude that applying a DAC to a ADC signal would result in your input signal. But does it?

Consider the information that is lost during these steps - can we reconstruct our input signal?

With a combination of these two devices, an attempt to reconstruct an undersampled signal can vary wildly, and even give an enveloped signal - meaning that the power of the reconstructed signal is transmitted with the "enclosing" wave of a different frequency. For example, attempting to sample a signal nearly twice your sampling frequency gives:
1.4 Reconstruction Filters

We can use filters to smooth over the discontinuities introduced by our DACs. What type of filter can do this? What are the advantages and disadvantages?