ELEC 3004 / 7312 – Systems: Signals & Controls **2017**

Problem Set 3: Digital Feedback Control

Total marks: 75 **Due Date:** Friday, June 2, 2017 at 23:59 AEST [end of week 13]

Note: This assignment is worth **20%** of the final course mark. Please submit answers via Platypus. Solutions, including equations, should be typed please and submitted directly in Platypus (preferred) or as PDF (n.b., Microsoft Word documents, scanned images of handwritten pages or items the clearly identify the author are specifically disallowed). The grade is determined by the teaching staff directly (which may be formed after peer reviews are entered). Finally, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions.

Thank you. **:-)**

Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. Answering an entire question with bare equations, lone numbers or without any verbal explanation is not acceptable. Marks may be reduced if an answer is of poor quality, demonstrates little effort or significant misunderstanding.

For Questions Q1 to Q4: Please answer **3 out of the 4** questions (your choice). If all four questions are submitted, *the tutors will only mark three chosen at random.*
Thus, of these, three questions should be answered.

For Question Q5: This question is optional and is for extra credit only.

Q1. State Space Delivers (Outputs) [25 points]

This problem considers the output of various state-space processes. For this please consider a LTI system given by:

$$\mathbf{A} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 0 & 1 \end{bmatrix} \qquad \mathbf{D} = [1]$$

where : $\mathbf{x}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\quad \mathbf{f}[n] = \mathbf{u}_{input}[n] = \mathbf{u}_{unit\,step}[n]$

(a) Please find the output in the Laplace Domain (i.e., $\mathbf{Y}(s)$)
[hint: $\mathbf{Y}(s) = \mathbf{C\Phi}(s)\,\mathbf{x}(0) + [\mathbf{C\Phi}(s)\,\mathbf{B} + \mathbf{D}]\,\mathbf{F}(s)$]

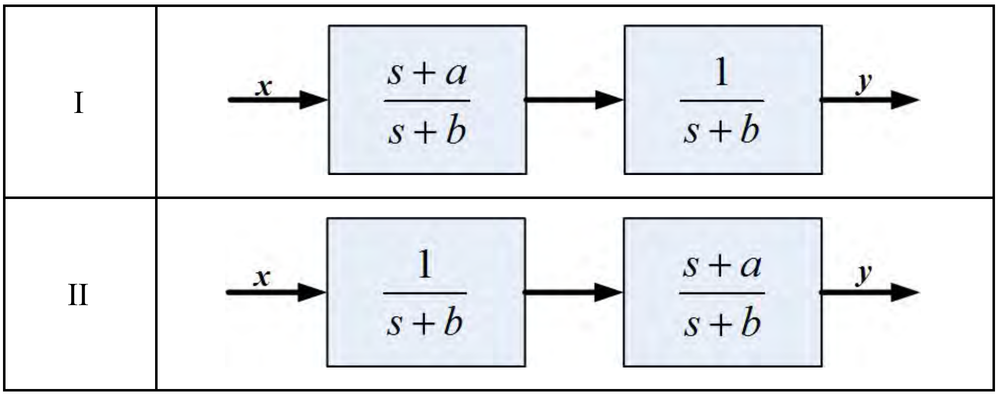
(b) Please find the output in the $\mathcal{Z}$-Domain (i.e., $\mathbf{Y}[z]$)
[hint: $\mathbf{Y}[z] = \mathbf{C}\left(\mathbf{I} - z^{-1}\mathbf{A}\right)^{-1} x[0] + \mathbf{H}[z]\,\mathbf{F}[z]$]

(c) Please find the output in the Discrete-Time Domain (i.e., $y[k]$)
[hint: $$\mathbf{y}[k] = \mathbf{CA}^k x[0] + \sum_{j=0}^{k-1} \mathbf{CA}^{k-1-j}\mathbf{Bf}[j] + \mathbf{Df}$$]

[**Textbook (Lathi) Reference Note**: This is based on Lathi, Question 13.6-1. The equations in the above hints are Eq. 13.37, 13.103 and 13.94 respectively]

Q2. An Order to Controllability and Observability **[25 points]**

This problem considers the influence (or lack thereof) that the order of operations has with regards to a system's controllability and observability. Consider two possible systems, I and II, as shown below



(a) What is the order of each system?

(b) What is the overall transfer function (i.e., $\frac{Y(s)}{X(s)}$) for each system?

(c) Determine the **Controllability** of each system?
[**hint:** please remember to also include whether (or not) the system is controllable]

(d) Determine the **Observability** of each system
[**hint:** please remember to also include whether (or not) the system is observable]

(e) Is Controllability and Observability order invariant for open-loop systems?
Please discuss.

[**Textbook (Lathi) Reference Note**: This is based on Lathi, Question 13.5-1]

Q3. LeviLab: Floating the (Magnetic) Data **[25 points]**

Laboratory 3 and 4 involve involve modelling and control of a levitating magnetic mass. Based on pre-lab analysis and your laboratory experiments, please answer the following:

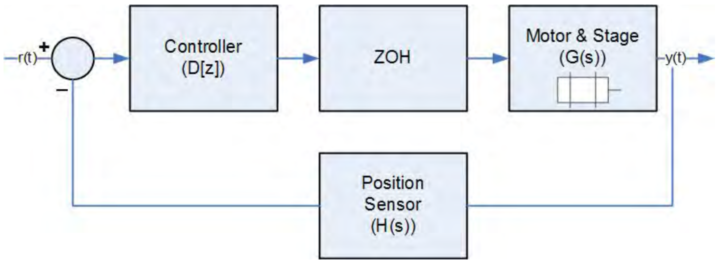
(a) **Give the system model**
That is, the **function / equation** describing the system's behavior . A set of differential equations or a state-space form is acceptable.

(b) **Tuning the system**
Discuss how you went about tuning the system. What gains did you select in the end? Please provide any evidence possible to support this (e.g. Ziegler Nichols process calculations, sensitivity plots, etc).

(c) **Did it levitate?**
Please provide the final control law (equation).
If you have it, please feel free to also include a picture of it levitating.

Q4. All the World's a Stage **[25 points]**

D. Bard of the *As You Like It* 3D Printer Company proposes a new design (for their new printer, the “The UpEnded”) in which the stage moves instead of the filament.
Its control is illustrated in the follow block diagram.



Chief Engineer Rosalind has selected a 100 Hz sample rate for the digital controller and characterised the plant (G(s)) as:

$$G(s) = \frac{1}{s\,(s+10)\,(s+20)}$$

(a) **Lead Compensator Design**
Assume that the controller is a Lead Compensator of the form
$$D(s) = \frac{K\,(s+a)}{s+b}$$
Rosalind would like a minimum performance of 5% Overshoot, a ≤0.5 second Settling Time and a ≤0.20 second Rise Time. Please find a series of gains (***K***, ***a***, ***b***) so as to meet this specification.

[**hint**: as a starting point, Celia (Rosalind’s assistant) gets a close (but not complete) solution of 5% Overshoot, 0.6 sec. Setting Time and 0.25 sec. Rise Time with the gains $K = 8000$, $a = 11$, $b = 62$]

(b) **Digital Compensator $\mathbf{D}\,[\mathbf{z}]$**
Please determine a digital compensator, $D\,[z]$ based on the compensator from part (a)
[hint: $D\,[z]$ may be thought of as being of the form $D\,[z] = C\frac{z-A}{z-B}$]

(c) **A PID-dling Option**
Touchstone instead suggests that a PID controller could be used instead. Such a controller would have the form:
$$D_{PID}\,[z] = K_P + \frac{K_I T z}{z-1} + K_D \frac{z-1}{Tz}$$
Should this be used? If so, please select and discuss which topology – P, PI, PD, or PID – should be used for this problem given Rosalind’s performance specifications. Please give and discuss an appropriate K_P, K_I, and K_D. If not, please justify.

(d) Ramping It Up
Please plot the magnitude response as a function of the (sample) time of the controller and overall system to a unit step and unit ramp input command (i.e., $r(t)$).

Q5. State-Space: Just Our Cup of Tea **[Upto + 15 points Extra Credit]**

Portia and Shylock want to serve the perfect cup of tea so as to expand their shop, *dx/dTea*, in Belmont (of course). Borrowing from the way espresso is made, Portia's idea is to pre-heat a tea cup by pouring boiling water ($\mathbf{T_{water}}$=100°C) into it and it drain it **t** seconds later, at which point the tea ($\mathbf{T_{tea}}$=90°C) is poured in immediately.
Shylock is worried about wasting perfectly good boiling water and wants to know the optimal time **t*** so as to maximize the Temperature of the tea when it first sipped ($\mathbf{T_{first\text{-}sip}}$). Assume $\mathbf{t_{first\text{-}sip}}$ to be at exactly 30 seconds after it is instantaneously poured. Shylock would like to know this time (**t***), temperature ($\mathbf{T_{first\text{-}sip}}$), and if it can be done with less hot water. (Bankers do not want to be in hot water, even should they serve it.)

To Tea or Not to Tea? This can be modelled using state-space. The temperature of water (or tea) in the cup is one state. Additional states are the temperature of the inside, middle, and outside of the cup. More generally, for the cup we can use an *n*-state finite element model. Then the vector $x(t) \in \mathbb{R}^{n+1}$ gives the temperature distribution across the fluid and the cup's continuum for a time **t**: where $x_1(t)$ is the water (or tea) temperature at time **t**, and $x_2(t), \ldots, x_{n+1}(t)$ is the temperatures of the elements of the cup.
This system's dynamics are then given by:

$$\frac{d}{dt}\left(x(t) - \mathbf{T_{env}}\right) = \mathbf{A}\left(x(t) - \mathbf{T_{env}}\right)$$

Where $A \in \mathbb{R}^{(n+1)\times(n+1)}$ and $\mathbf{T_{env}}$ is a vector of ***n*** scalar components at a value of the ambient temperature ($T_{ambinent}$).

Now at time $t = \mathbf{t}^*$ the liquid in the cup will change (immediately) from whatever value it has to 90°C (because this is the temperature of the tea being poured in). The other states (i.e., the temperatures of the cup's elements) will not change at that instant **t***. An example dynamics matrix, **A**, for ***n***=10 is given in `teamodel.m` (below and online).

[**Assume** that $T_{ambinent}$=22°C, Temperature & time resolution of 0.1 seconds]

(a) **Optimal pre-heating time t* and Maximum $\mathbf{T_{first\text{-}sip}}$**
Please determine and justify the time **t*** and its **first-sip tea temperature**.
As part of this justification please explain the method, submit the code, provide any supporting graphs and give final answer result (i.e., the resulting maximum first-sip tea temperature, $T_{first\text{-}sip}$ for the optimal value of t*).

(b) **Testing the Waters**
To understand the impact of heating the cup, please analyse and plot $\mathbf{T_{first\text{-}sip}}$ against the time after pouring (by varying $\mathbf{t_{first\text{-}sip}}$) with and without Portia's "pre-heating."

(c) **Don't Bleed Shylock Dry**
Hot water is expensive (in both energy and time). What if better cups were used? Find an new **A**, such that $\mathbf{t_\eta}$***** is ½**t*** from above. **THEN**, what if "warm" pre-heat water ($\mathbf{T_{warm\text{-}water}}$=70°C) is used instead, what would $\mathbf{t_{\eta\text{-}warm}}$* and $\mathbf{T_{\eta\text{-}warm\text{-}first\text{-}sip}}$ be?

[**Reference Note**: Based on material from S. Boyd, *Lecture Notes for EE263*, 2012]

Appendix: Q5 Initial Value Matlab Code [teamodel.m]

```
%% ELEC3004 - Problem Set 3 - Question 5 - INITIAL DATA
% Data for "State-Space: Just Our Cup of Tea"
% Based on espressodata.m by Stephen Boyd, Lecture Notes for EE263, 2012
% (§ EE263 homework problems, p. 88)

%% Initial Parameters
% Temperature in degrees C
% Time in seconds
n      = 10;   % Number of Elements.
              % Feel free to update!  Though if you do you will need to change A!
Ta   = 22;   % Ambient temperature
Tl   = 100;  % Temperature of pre-heat water
Te   = 90;   % Temperature of tea
tfs  = 30;   % Time after pouring tea for first sip of tea (in seconds)
tres = 0.1;  % Time discretization/resolution (0.1 seconds)
Tres = 0.1;  % Temperature discretization/resolution (0.1°C)
A = [...     % The Tea Cup's System Model
 -1.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00;
 33.33 -44.44  11.11   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00;
  0.00  11.11 -22.22  11.11   0.00   0.00   0.00   0.00   0.00   0.00   0.00;
  0.00   0.00  11.11 -22.22  11.11   0.00   0.00   0.00   0.00   0.00   0.00;
  0.00   0.00   0.00  11.11 -22.22  11.11   0.00   0.00   0.00   0.00   0.00;
  0.00   0.00   0.00   0.00  11.11 -22.22  11.11   0.00   0.00   0.00   0.00;
  0.00   0.00   0.00   0.00   0.00  11.11 -22.22  11.11   0.00   0.00   0.00;
  0.00   0.00   0.00   0.00   0.00   0.00  11.11 -22.22  11.11   0.00   0.00;
  0.00   0.00   0.00   0.00   0.00   0.00   0.00  11.11 -22.22  11.11   0.00;
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00  11.11 -22.22  11.11;
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00  11.11 -11.31;
];
```