ELEC 3004 / 7312 – Systems: Signals & Controls **2017**

Problem Set 1: An Introduction to Signals and Systems

Total marks: 100 **Due Date:** March 25, 2017 at 23:59 AEST [end of week 4]

Note: This assignment is worth **20%** of the final course mark. Please submit answers via Platypus. Solutions, including equations, should be typed please and submitted directly in Platypus (preferred) or as PDF (n.b., Microsoft Word documents and/or scanned images of handwritten pages are specifically disallowed). The grade is determined by the teaching staff directly (which may be formed after peer reviews are entered). Finally, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions.

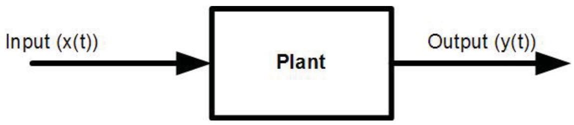
Thank you. **:-)**

Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. Answering an entire question with bare equations, lone numbers or without any verbal explanation is not acceptable. Marks may be reduced if an answer is of poor quality, demonstrates little effort or significant misunderstanding.

Q1. Planting a First Step **[20 points]**

Consider an open loop system with an input signal ($x(t)$), a plant with a given transfer function and an output signal ($y(t)$). In this problem $u(t)$ is the unit step function.



For each pair of input and output systems below

- Plot the input and output signals[1].
- State whether the system is linear, and why.
- State whether the system is causal, and why.

  [Please recall that $u(t)$ is the unit step function]

(a) $x(t) = u(t) + 2u\left(t-2\right)$

$y(t) = u(t)\left(1-e^{-t}\right) + u(t-2)\left(2-2e^{-(t-2)}\right)$

(b) $x(t) = u(t) - u\left(t-2\right)$

$y(t) = u(t)e^{t}\sin(50\pi t) - u(t-2)e^{t-2}sin(50\pi(t-2))$

(c) $x(t) = u(t) + u\left(t-1\right)$

$y(t) = u(t)\left(1-e^{-t}\right) + u(t+1)\left(1-e^{-(t+1)}\right)$

(d) $x(t) = u(t) + 2u\left(t-1\right)$

$y(t) = u(t)\left(1-e^{-t}\right) + u(t-1)\left(1-e^{-(t-1)}\right)$

[1] In MATLAB the '`step()`' and/or '`heaviside()`' functions may be helpful. See also `doc step` or `doc heaviside` for more information.

Q2. Sampling: Improving the Audio(s) to Noise Ratio? [20 points]

Eleda has a new song, *Adiós*, a highlight of which is a loud Soprano C note (**1046.50 Hz**). This was recorded live at her recent concert at Sheepish Stadium via a microphone connected to a preamp that approximates a consumer line level signal.

Upon inspection the signal recorded was found to be (in Volts):
$V_{microphone}(t) = 0.447\cos(2093\pi t) + 0.314\cos(100\pi t) + 1$

It appears that joint between the 3.5 mm connector and the unbalanced wire was not properly shielded and thus introducing a 50 Hz whine. To add insult to injury, the secretive producer (Off-the-Record Records) only sampled this at **2,205 Hz** (instead of the expected 44.1 KHz).

(a) Please plot the Voltage signal from microphone ($V_{microphone}(t)$) for t=0 to 1 second.

(b) Please plot the sampled, digitized signal captured on a basic audio card with simple line level (i.e., no negative voltage rail). Again, for t=0 to 1 second.
[hint: a line level signal is basically an alternating current signal with a DC offset]

(c) It is proposed that all this can be solved "easily" by changing the anti-aliasing (or band-limiting) filter to add a high-pass filter with a cut-off of 100 Hz between the pre-amp and the line-level input on the audio card. Briefly discuss if this will work?
[hint: it might help to generate a plot of the signal with this filter]

(d) Suppose that our singer catches a cold, and rather than producing a 1046.50 Hz sine wave, she produces a raspy 1046.50 Hz square wave. Does the sampled waveform change? If it changes, briefly detail how. If it stays the same, briefly explain why.

Q3. A Convoluted Question [20 points]

This question is about the mathematical process of convolution. Suppose we have some discrete linear system, and we give it an impulse input. That is: `[1, 0, 0, 0, 0]`. We measure the output of the system, and we get back: `[1, 3, 2, 1, 0]`.

As was mentioned (in class) now that we know the impulse response, we can calculate the output response of the system due to any arbitrary input.

(a) Under what assumptions is the aforementioned statement true?
[hint: In other words, please let under which assumptions will this process work.]

(b) Please find the output response to the step input response of
$\mathbf{x}_{step}$=`[0, 0, 1, 1, 1, 1, 1, 1, 1, 1]` and
$\mathbf{x}_{ramp}$=`[1, 2, 3, 4, 5, 6, 6, 6, 6, 6, 6]`.

(c) If the output response is: `[1, 3, 3, 4, 3, 4, 2, 1, 0, 0]`,
what was the input response?[2]

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[2] What is the opposite of convolution?

Q4. A New Basis **[20 points]**

Remember that a signal (vector) need not only be written in the standard basis (**S** consisting of basis vector $\mathbf{s}_1,\ldots,\mathbf{s}_n$, where $\mathbf{s}_i$ are columns of an identity matrix (i.e., **S**=**I**))

While every vector $\boldsymbol{v}$ in $\mathbb{R}^n$ can be written in exactly one way as a combination of the basis vectors, a new set of basis vectors may be chosen or designed. One, of many, convenient basis is the Wavelet basis. Wavelets are "small waves" (from the French *ondelette*) that have different lengths and are located at different places and where each basis vector has more "frequency."

Recall from linear algebra that for a set of vectors ($\boldsymbol{w}$) to form a basis in $\mathbb{R}^n$ that:

1. The vectors ($\mathbf{w}_1,\ldots,\mathbf{w}_n$) are linearly independent
2. These basis vectors may be seen as columns and assembled to form a n×n basis matrix (**W**) that is invertible
3. Related to the aforementioned note, a given vector ($\boldsymbol{x}$) may be written as a linear combination of coefficients ($\boldsymbol{c}$) such that $\boldsymbol{x}=\mathbf{W}\boldsymbol{c}$ as:

$$\mathbf{x} = c_1\mathbf{w}_1 + c_2\mathbf{w}_2 + \ldots + c_n\mathbf{w}_n$$

(a) Consider $\mathbf{x} = [4, 0, 7, 2]$.
For this case, what are the standard basis vectors $\mathbf{s}_1,\ldots,\mathbf{s}_4$?
Please determine the scalar coefficients $\mathbf{c}_1,\ldots,\mathbf{c}_4$?

(b) Consider a small (4×4) wavelet basis given by:

$$\mathbf{w}_1 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix}1\\1\\-1\\-1\end{bmatrix}, \mathbf{w}_3 = \begin{bmatrix}1\\-1\\0\\0\end{bmatrix}, \mathbf{w}_4 = \begin{bmatrix}0\\0\\1\\-1\end{bmatrix}$$

Using this basis, determine the basis matrix **W** and its inverse $\mathbf{W}^{-1}$
[**Hint**: you can do this by hand, consider the properties of **W**]

Now, please find the coefficients for the vectors:

- $\mathbf{x_1} = [4, 0, 7, 2]$
- $\mathbf{x_2} = [8, 6, 4, 2]$
- $\mathbf{x_3} = [-1, 1, 0, 0]$
- $\mathbf{x_4} = [\text{The last four digits of your student number}]$

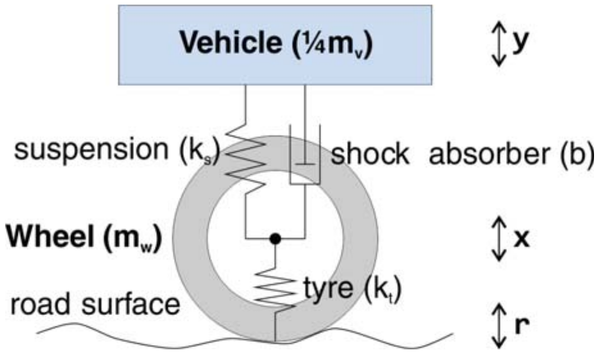
(c) For the case (b) above, it has been postulated that the coefficients, **c**, should always be given by $\mathbf{c} = \mathbf{W}^{-1}\mathbf{x}$. Please prove or disprove this.

(d) For case (b), how does the coefficient c_1 relate to the values $x_1,\ldots,x_4$?

Q5. Applications: LCCODEs: Suspension of Belief?

[20 points]

Consider the vehicle suspension of a four-wheel car as it drives over a road surface. This may be modeled as two masses: $m_1 = m_{wheel}$ as the mass of the wheel and $m_2 = \frac{m_v}{4}$ as one quarter the mass of the vehicle body. We assume linear springs for the suspension and tyre and viscous damping.



(In automotive engineering this is called the "quarter-car model," wherein m_1 is the "unsprung mass" and m_2 assumes homogeneous weight distribution for the car. The latter is not entirely the case for most cars, which are "front-heavy" so as to "understeer" [or "phase-lag"], especially at high speed).

(a) What is the LCCODE that describes the vehicle with regards to the road (i.e., $\frac{y}{r}$)?
[hint: this may be left in the Laplacian, i.e., $\frac{Y(s)}{R(s)}$]

(b) Assuming $m_v = 1000kg$, $m_w = 10kg$, $k_s = 10\frac{kN}{m}$, $k_t = 1\frac{kN}{m}$,and $b = 5\frac{kN}{ms}$ (these are completely **unrealistic**, but simple values), please solve this LCCODE. What are the principal oscillation frequencies (often denoted **ω**) of this system if it were to hit a 10 cm high step-high speed bump?
[hint: for fun, please look up the values for your own car]

(c) What happens to oscillation frequencies in (b) if wheel is lighter (e.g., made from carbon fibre) such that $m_w = 1kg$?

(d) Bottoming Out!
Any real suspension is limited in displacement. Let us define $d = y - r$ as this suspension displacement (compression). Clearly if d is "too much" the suspension hits a hard stop. At that point, this LCCODE model is also invalid.
To prevent this, what is the maximum height C_{max} (in meters) that the suspension can handle without hitting the hard stop (i.e., "bottoming out")?

Correction (March 20): One could solve this for an arbitrary d_{max}. However, if it helps let us define $d_{max} = 0.1m$ thus, the simple limit $|d(t)| > 0.1m$ is for "bottoming out".