Questions

Q1. [10 points]
Consider the following circuit in which \( R = 5 \Omega \), \( L = 1 \text{H} \) and its transfer function \( H(s) \).

1. Solve a mesh equation to find the time-domain current of this circuit in response to the following \( V_{\text{in}} \). [2 points]
   \[
   V_{\text{in}} = \delta(t)
   \]
   The time-domain current is the circuit’s Impulse Response\(^2\). How does it relate to the circuit’s transfer function? [1 point]

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1 This is the [Dirac delta function](http://example.com/DiracDelta).
2 This need not always be a current. We have simply defined the circuit’s output to be the mesh current.
2. What is the time-domain current of this circuit in response to the following $V_{in}$? [1 point]

$$V_{in} = \delta(t - 1)$$

How is this related to the current derived in (1) and what property of the system does this demonstrate? [1 point]

3. Without doing any calculations, can you predict the time-domain current of this circuit for the following $V_{in}$? [1 point]

$$V_{in} = 2 \cdot \delta(t) + 5 \cdot \delta(t - 2)$$

Justify your answer using principles and properties of the system (the circuit). [4 points]

Q2. [5 points]

In Question 1, part 3 you observed a circuit’s response to a sum of time-shifted impulses. In fact, every signal is a sum of time-shifted impulses. The following integral states this fact mathematically:

$$\int_{-\infty}^{\infty} \delta(t - T)f(T)dT = f(t)$$

1. Given this fact, construct an integral that could be used to evaluate a circuit’s response to an input $f(t)$, given the circuit’s impulse response $H(t)$ [1 point].

Such an integral would be of the form below:

$$F_{system}(f(t)) = \int_{-\infty}^{\infty} \ldots \ldots \ldots dT$$

2. Discuss why the integral produces the system output. [4 points]

Hint: You will replace the …… with some expression involving $H(t)$ the impulse response and $f(t)$ the input. You may think of an integral as a “continuous sum”.
Q3. [5 points]
Try solving the next circuit using superposition (do not consider V2 in your superposition).

1. Present your superposition calculations briefly. Pages of equations should not be necessary. [1 point]
2. Briefly calculate the output voltage without using superposition and compare it to the voltage from (1). [1 point]
3. Explain your results, citing and applying principles you deem relevant. Does superposition give the correct result? Why? Based on your superposition result, do you think the above system is linear or nonlinear? [3 points]
Q4. [5 points]
Suppose the resistance of a copper motor winding increases by 1Ω per second during periods of extremely high current. We’d like to see how hard this is to model.

1. Derive a time-domain expression for Vout / Vin as a function of time from t = 0 onwards. Do not go into the s-domain. [1 point]
2. Find the transfer function H(s) of this circuit. Now solve in the s-domain for Vout with a unit step input. [1 point]
3. Does your s-domain solution agree with your solution from (1)? Why do you think this is the case? [3 points]

Q5. [0 points – For Review :-) ]
True or False:

1. If a system is linear, it can be written as f(x) = Ax
2. Given the canonical form (\( \dot{x} = Ax + Bu \), etc.), if \( B = 0 \), then system is autonomous.
3. A Linear Time Invariant (LTI) system must have constant system, control, etc. matrices (A, B, C, and D in canonical form).
4. The two requirements for a system to be linear are (1) Additivity and (2) Homogeneity
5. A capacitor is an example of an invertible system
6. All real-time systems must be causal
7. The inverse of a first-difference operation (\( y[n] = x[n] - x[n-1] \)) is the accumulator

\( y[n] = \sum_{k=-\infty}^{n} x[k] \).

Imagine two systems in cascade:

8. If systems A and B are LTI, then so is the overall system
9. If systems A and B are non-linear, then so is the overall system
10. If systems A and B are stable, then so is the overall system