ELEC 3004 / 7312 – Systems: Signals & Controls **2019**

Problem Set 1: An Introduction to Signals and Systems

Total marks: 100 **Due Date:** Sunday, **April 7, 2019** at 23:59 AEST [*end of week 6*]

Note: This assignment is worth **20%** of the final course mark. Please submit answers via Platypus. Solutions, including equations, should be typed please and **submitted directly in Platypus** (preferred) or as **PDF**. Note that Microsoft **Word** documents and scanned images of **handwritten pages** are specifically **disallowed**. The grade is determined by the teaching staff directly (which may be formed after peer reviews are entered). Please double-check that your name is not in the solution directly or via the associated metadata.

Also, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions.

Finally, a note of remembrance for our colleagues and compatriots in Christchurch.
For, no matter the signal, we are but one system -- United.

Thank you. **:-)**

Questions

Explain your solutions as if you are trying to **teach a peer**. Demonstrate your insight and understanding. Answering an entire question with bare equations, lone numbers or without any verbal explanation is not acceptable. Marks may be reduced if an answer is of poor quality, demonstrates little effort or significant misunderstanding.

Q1. Linearity: Starting Straight Away [10 points]

This question explores some of these interesting properties of Linear systems, notably superposition. This gives that if several inputs are acting on a linear system, then the total response of this system is the sum of the outputs from each input on its own.

Please **determine** and **justify** if these equations are linear

(a) $w(t) = 300t + 4$

(b) $y(t) = 300\frac{dx}{dt} + 4x(t)$

(c) $z(t) = 300 \cdot t \cdot \frac{dx}{dt} + 4 \cdot t^2 \cdot x(t)$

Please **determine** and then **generally prove** (or **disprove**) if these statements about linearity are true.

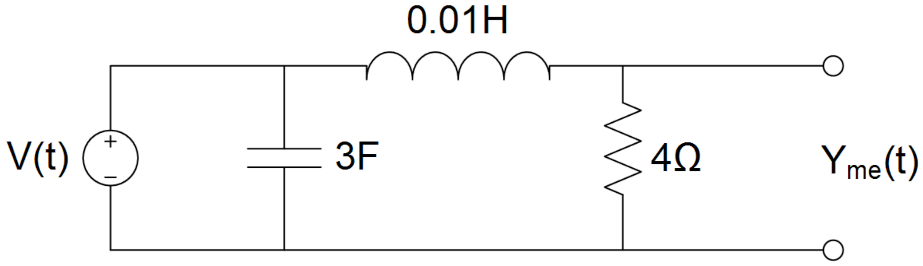
(d) Linearity and the Converse. Consider $f(x) = [A]x$.
For this case, is matrix multiplication a linear operation?
The Converse: Can **any** linear function f **always** be written as $f(x) = [A]x$?

(e) **Uniqueness**.
For any linear function f there is **only** one matrix $[A]$ for which $f(x) = [A]x$ for all x.

Q2. Charging Ahead! **[10 points]**

Let us consider the interplay between linearity, circuits and signals.

A Magic Elf proposes the following following circuit consisting of ideal elements:



(a) Is the voltage output for this circuit (noted Y_{me}(t)) a linear system?

(b) If so, show that the output of the Magic Elf's circuit, Y_{me}(t), satisfies the conditions of linearity with respect to the input, V(t) and the initial conditions.

If not, what element(s) could be removed to make it linear?

Q3. Sampling: Dreaming of Less Noise **[15 points]**

Pink has a new song, *Noise*, a highlight of which is a loud Mezzo-soprano A_5 note (**880 Hz**). This was recorded live at the recent concert at 1E6 Dreams† Stadium via a microphone connected to a preamp that approximates a consumer line level signal.

[†: 1 million dreams is 2,740 person·years at 1 dream/day]

Upon inspection the signal recorded was found to be (in Volts):

$V_{microphone}(t)=0.42\cos(1760\boldsymbol{\pi}t)+0.314\cos(100\boldsymbol{\pi}t)+1$

It appears that joint between the 3.5 mm connector and the unbalanced wire was not properly shielded and thus introducing a 50 Hz whine. To add insult to injury, the recording was rushed to get ahead of a demolition for refurbishment, so by accident it was sampled at **1,044 Hz** (instead of the expected 44.1 KHz).

(a) Please plot the voltage signal from the microphone ($V_{microphone}(t)$) for t=0 to 1 second.

(b) Please plot the sampled, digitized signal captured on a basic audio card with simple line level (i.e., no negative voltage rail). Again, for t=0 to 1 second.
[hint: a line level signal is basically an alternating current signal with a DC offset]

(c) It is proposed that all this can be solved "easily" by changing the anti-aliasing (or band-limiting) filter to add a high-pass filter with a cut-off of 100 Hz between the pre-amp and the line-level input on the audio card. Briefly discuss if this will work?
[hint: it might help to generate a plot of the signal with this filter]

Q4. Sampling's Rhythm: Got It Beat **[15 points]**

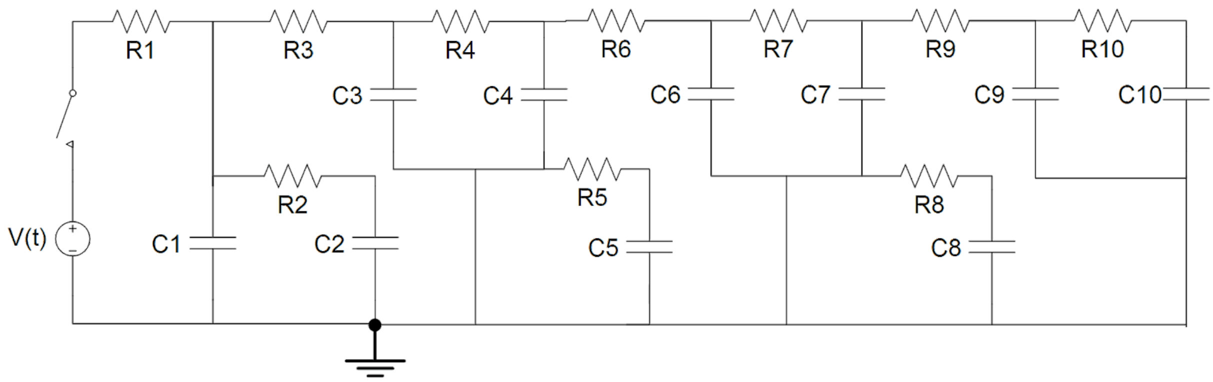
Let ***f(t)*** be a periodic continuous time signal with Period **P**. Then, let **f[k]** be the discrete time signal generated from ***f(t)*** with equally spaced samples of period **Q**; that is,

$$\mathbf{f[k] = f(kQ)}$$

Show that the sequence **f[k]** will be periodic **if and only** if the ratio **P/Q** is itself rational.

Q5. The 10-Channel Project: A Rather Dynamical Circuit **[30 points]**

An interconnect circuit is being considered as part of a new logic architecture.



Initially, assume unit circuit elements. That is, the capacitors $C_1,..,C_{10}$=1F, the resistors $R_1,..,R_{10}=1\Omega$ and voltage, V(t)=1 V. At t=0, the switch is closed.

The voltages across the capacitors (C[i], here C1 to C10) are represented as $\mathbf{x_i}$.

Hint: V(t) may be seen as an input/control signal (sometimes called u(t)). Then, a LDS model for this circuit could be in the form $\dot{x} = [A]x + [B]u$, $y = x$. In which case, $exp(tA)x(0)$ would the autonomous response and $inv(sI - A) \cdot B$ is the transfer matrix (where $inv(sI - A)$ is the resolvent).

(a) What are the steady-state values (i.e., the static gain matrix) of the voltages across the capacitors in this circuit. That is, what are the final values of $x_1,...,x_{10}$?

(b) Let's write this system as an LDS model. What are the [A] and [B] matrices? What are the eigenvalues and resolvent of [A]. And, importantly, what do the **eigenvalues**, **transfer matrix**, and **QR** [or Gram-Schmidt] **decomposition** indicate/signify about the system?

(c) Which of the ten voltages will reach within 99% of its steady-state values last? How long will it take for it (and thus the circuit) to reach within 99% of these final values?

(d) Please plot the step response of the system. That is, please plot the voltages (or states) $x_1,...,x_{10}$ as a function of time from 0 to the settling time found in (b)

(e) What is the effect of doubling the voltage on the overall settling time? Based on this, would it be possible to select a voltage such that the system will deliver 1 Volt across C10 in t=0.3004s (i.e., x_{10}(t=0.3004s) = 1 Volt).

Q6. A New Basis **[20 points]**

Remember that a signal (vector) need not only be written in the standard basis (**S** consisting of basis vector $\mathbf{s_1},\ldots,\mathbf{s_n}$, where $\mathbf{s_i}$ are columns of an identity matrix (i.e., **S**=**I**))

While every vector $\boldsymbol{v}$ in $\mathbb{R}^n$ can be written in exactly one way as a combination of the basis vectors, a new set of basis vectors may be chosen or designed. One, of many, convenient basis is the Wavelet basis. Wavelets are "small waves" (from the French *ondelette*) that have different lengths and are located at different places and where each basis vector has more "frequency."

Recall from linear algebra that for a set of vectors ($\boldsymbol{w}$) to form a basis in $\mathbb{R}^n$ that:

1. The vectors ($\mathbf{w_1},\ldots,\mathbf{w_n}$) are linearly independent
2. These basis vectors may be seen as columns and assembled to form a n×n basis matrix (**W**) that is invertible
3. Related to the aforementioned note, a given vector ($\boldsymbol{x}$) may be written as a linear combination of coefficients ($\boldsymbol{c}$) such that $\boldsymbol{x}=\mathbf{W}\boldsymbol{c}$ as:

$$\mathbf{x} = c_1\mathbf{w_1} + c_2\mathbf{w_2} + \ldots + c_n\mathbf{w_n}$$

(a) Consider **x**=`[1,10,100,1000]`.
For this case, what are the standard basis vectors $\mathbf{s_1},\ldots,\mathbf{s_4}$?
Please determine the scalar coefficients $\mathbf{c_1},\ldots,\mathbf{c_4}$?

(b) Consider a small (4×4) wavelet basis given by:

$$\mathbf{w_1} = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \mathbf{w_2} = \begin{bmatrix}1\\1\\-1\\-1\end{bmatrix}, \mathbf{w_3} = \begin{bmatrix}1\\-1\\0\\0\end{bmatrix}, \mathbf{w_4} = \begin{bmatrix}0\\0\\1\\-1\end{bmatrix}$$

Using this basis, determine the basis matrix **W** and its inverse $\mathbf{W}^{-1}$
[**Hint**: you can do this by hand, consider the properties of **W**]

Now, please find the coefficients for the vectors:

- $\mathbf{x_1}$=`[1,10,100,1000]`
- $\mathbf{x_2}$=`[1000,100,10,1]`
- $\mathbf{x_3}$=`[3,5,3,2]`
- $\mathbf{x_4}$=`[3,0,0,4]`
- $\mathbf{x_5}$=`[3,1,4,1]`
- $\mathbf{x_6}$=`[1,-1,0,1]`

(c) For the case (b) above, it has been postulated that the coefficients, **c**, should always be given by $\mathbf{c} = \mathbf{W}^{-1}\mathbf{x}$. Please prove or disprove this.

(d) For case (b), how does the coefficient c_1 relate to the values $x_1,\ldots,x_4$?