THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

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Student Number |__|__|__|__|__|__|__|__|__|

Family Name ______________________

First Name ______________________

School of Information Technology and Electrical Engineering

EXAMINATION

Semester One Final Examinations, 2018

ELEC3004 Signals, Systems and Control

This paper is for St Lucia Campus students.

Examination Duration: 180 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - specified materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Casio FX82 series or UQ approved (labelled)

One A4 sheet of handwritten notes double sided is permitted

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

1 x 1cm x 1cm Graph Paper

Instructions To Students:

Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.

All answers should be written in the answer booklet provided. You MUST show the steps used to arrive at your final solution. Best marks will be awarded to numerically correct solutions showing complete working. Partial credit will be given to partially complete solutions or to incorrect solutions where errors have been carried through calculations. Assumptions, if any, should be explicitly stated and justified/verified as appropriate.

For Examiner Use Only

Question	Mark
1	
2	
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11	

Total ________

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This exam has TWO (2) Sections for a total of 100 Points

Section 1: **Signals and Systems**...60 Points
Section 2: **Control** ....40 Points

⇨ Please answer **ALL** questions + **ALL Answers MUST Be Justified** ⇦
(answers alone are **not** sufficient)

⇨ **PLEASE RECORD ALL ANSWERS IN THE ANSWER BOOKLET** ⇦

(Any material not in Answer Booklet(s) **will not be seen**.
In particular, the exam questions paper **will not be graded** or reviewed.)

Section 1: Signals and Systems

Please Record Answers in the **Answer Booklet** (**8 Questions | 60 Points**)

1. Briefly give THREE advantages of digital signals over analogue signals and TWO advantages of analogue signals over digital signals?

(5 Points)

2. Given a discrete-time **unit impulse response** with the following difference equation:

$$\boldsymbol{y[n] = 3 \cdot \delta[n - 4] - \delta[n + 0] + \delta[n + 7] + 4 \cdot \delta[n + 2]}$$

A. What is its Z-transform?
(i.e., what is **Y(z)**)?

B. What is its frequency (or Fourier) response?
(i.e., what is **Y(ω)**)
[hint: for partial credit, you may leave it in terms of the phasor $e^{j\omega}$]

(5 Points)

3. You may recall the "Wagon Wheel Effect" optical illusion

A. Explain, in terms of the Sampling Theorem, why the wheel of a car in a movie can appear to turn the wrong way. What signal processing is performed by the brain when analogue motion is perceived while viewing a correctly projected motion-picture?

B. Typical "HD Cinema" is 24fps progressive scan. If we assume a global shutter, how fast does a wheel need to turn (in revolutions/minute (rpm)) so as to appear to go backwards.

(5 Points)

4. The frequency response magnitude of a normalised Butterworth filter of order n is given by:

$$|H_n(\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}}$$

   A. Please determine the transfer function of a 2^{nd}-order, **high-pass** Butterworth filter with cut-in frequency equal to 6 kHz.

   B. At what frequency is the gain of this filter $-2dB$? $-20dB$?

(5 Points)

5. Consider the following input sequence to a linear time invariant system

$$\{x[n]\}_0^4 = \{0,3,1,0,-1\}$$

   A. Write the difference equation for $x[n]$ in terms of the impulse function $\delta[n]$.
   B. Write the output sequence $y[n]$ in terms of the impulse response function $h[n]$.
   C. Hence, show that the output $y[n]$ of a linear time invariant system is a discrete convolution of the input sequence $x[n]$ and the impulse response function $h[n]$.

(5 Points)

6. For the sequence

$$x[n] = \{0,3,0,0,4,0,-7,1,0\}$$

And, an **up-sampling** factor of $L = 2$, please reconstruct the output sequence $x(t)$ via:

   A. A Zero Order Hold (ZOH) interpolation approach

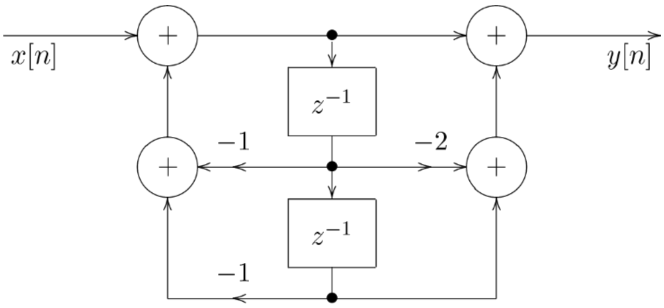
   B. A Whittaker–Shannon ($sinc$) interpolation approach

   [**Hint**: Presenting our answer as either a function **or** a plot is acceptable.]

(10 Points)

(Questions Continue Overleaf)

7. Consider the digital filter given by:



For this filter

A. Determine if it is an IIR or FIR

B. Calculate the difference equation

C. Calculate the first four (4) samples of the impulse response

D. Calculate the gain at DC (i.e., $\omega = 0$)

E. Calculate the gain at the Nyquist frequency (i.e., $\omega = \frac{\omega_s}{2}$)

(10 Points)

8. Inspired by an Irish ballad, Mr Crooner plans to quit his job and form the band Me2. He plans to record his songs digitally to compact disc (CD). Assume that the songs have frequencies ranging from 100 Hz-1,100 Hz (i.e. 1.1 kHz).

A. In this case, what is the Nyquist rate?

B. Briefly, what is an assumption about the clock/sampler for digital signals?

C. A CD actually uses 44,100 samples/s. If the samples are quantized into 65,536 levels ($L = 65{,}536$), determine the number of binary digits required to encode a 3:24 (3 minute and 24 second) long song?

D. Mr. Crooner's agent, Stephen Cacophonous, has no time for so many samples and wants to record songs using 2,000 samples/sec instead.
What will happen? Will there be aliasing? If so, what frequencies will alias?

E. Besides adding an additional anti-aliasing filter, suggest a way that aliasing (if it exists) can be removed from the above case (in D) without changing the sample rate (from 2,000 samples/sec).

(15 Points)

(Questions Continue Overleaf)

Section 2: Control

Please Record Answers in the **Answer Booklet** (**3 Questions | 40 Points**)

9. Consider an electric motor with shaft angular velocity ω. The motor is controlled by input voltage $u(t) = \varepsilon + Ri$. The equations of motion for the system are:

$$\varepsilon = \lambda\omega$$
$$\tau = \lambda i$$
$$\mathrm{I}_r\dot{\omega} = \tau$$

where $\mathrm{I}_r = 1.35 \times 10^{-4}$ Nm/rad/s2 is the rotational inertia of the rotor, τ is the output torque of the motor in Nm, $\lambda = 795 \times 10^{-3}$ V/rads-1 is the flux-linkage coefficient of the motor and $R = 0.01\Omega$ is the motor internal resistance.

A. Derive a discrete transfer function from $u(t)$ to ω for this plant, using the matched pole-zero method. Draw the system's poles and zeros, if any, on the z-plane.

B. How will the system's dynamic response change under unit feedback, if the sample period is gradually increased? Draw the approximate path on the z-plane diagram in part a.

(10 Points)

10. A quadrotor drone manoeuvers by changing thrusts on opposite pairs of rotors. Consider a simple planar linear model of a quadrotor's pitch angle dynamics:

$$\mathrm{I}\ddot{\theta} = 2dk(\Delta\omega(t))$$

Where θ is the pitch angle, I is the vehicle's rotational inertia, d is the rotor arm length, k is an aerodynamic constant, $\Delta\omega(t)$ is an instantaneous rotor velocity delta (the control parameter).

A. Derive a continuous transfer function of the plant, from $\Delta\omega(t)$ to θ

B. Digitise the continuous transfer function for sample time T using Euler's method.

C. Plot the transfer function's poles on the z-plane and draw the system root locus under closed loop feedback.

D. Sketch the positions of the poles and zeros of a digital PD controller that would stabilise pitch angle under feedback control, and draw the approximate path of the closed loop pole positions with increasing gain.

E. Sketch the positions of the poles and zeros of a digital PD controller that would stabilise pitch angle under feedback control, this time incorporating the extra dynamics of the motor system described in question 1, and draw the approximate path of the closed loop pole positions with increasing gain.

F. How should the PD controller be changed to accommodate the added motor dynamics?

(20 Points)

11. Political Signals
The despot known as “Shekhar the Horrible" rules the land with an iron fist.

Four powerful factions support Lord Chandra: the Illuminati, the Cryptofascists, the Central Administration and the People for the Ethical Treatment of Academics. These groups determine their support for Lord Chandra's regime based on the daily newspaper they each receive, basing their opinions on other rival factions' reported opinions. Chandra will be overthrown if the combined opinions of the Illuminati and the Cryptofascists exceeds some very large (but finite) value of dissatisfaction*. Under Lord Shekhar's corrupt rule, the daily newspaper has become unreliable and delays are expected. The People for the Ethical Treatment of Academics and the Central Administration both receive their newspapers a day late. The Illuminati's and Crypofascists' newspapers are not delayed.

After receiving their daily papers:
- The opinion of the Illuminati's will be half that of the Central Administration.
- The Cryptofascists opinion will be the same as the People for the Ethical Treatment of Academics'.
- The People for the Ethical Treatment of Academics' opinion will be the same as the Illuminati's opinion (from yesterday).
- The opinion of the Central Administration will be the negative of the Cryptofascists' opinion (from yesterday).

(*It doesn’t matter how satisfied the constituents ever become - nobody cares that you had a parade in your honour yesterday or tomorrow, if you're deposed and beheaded today.)

A. (10%) Will Shekhar the Horrible's dire reign last forever? Prove your answer analytically.
B. (10% extra credit) Using part a, comment on the importance of timely public infrastructure to stable governance.

(10 points + 10 marks extra credit)

END OF EXAMINATION

(See Tables Overleaf)

Table 1: Commonly used Formulae

The Laplace Transform

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt$$

The $\mathcal{Z}$ Transform

$$F(z) = \sum_{n=0}^{\infty} f[n]z^{-n}$$

IIR Filter Pre-warp

$$\omega_a = \frac{2}{\Delta t}\tan\left(\frac{\omega_d \Delta t}{2}\right)$$

Bi-linear Transform

$$s = \frac{2(1-z^{-1})}{\Delta t(1+z^{-1})}$$

FIR Filter Coefficients

$$c_n = \frac{\Delta t}{\pi}\int_0^{\pi/\Delta t} H_d(\omega)\cos(n\omega\Delta t)\,d\omega$$

Table 2: Comparison of Fourier representations.

Time Domain	***Periodic***	***Non-periodic***	
Discrete	***Discrete Fourier Transform*** $$\tilde{X}[k] = \frac{1}{N}\sum_{n=0}^{N-1}\tilde{x}[n]e^{-j2\pi kn/N}$$ $$\tilde{x}[n] = \sum_{k=0}^{N-1}\tilde{X}[k]e^{j2\pi kn/N}$$	***Discrete-Time Fourier Transform*** $$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$ $$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)e^{j\omega n}\,d\omega$$	***Periodic***
Continuous	***Complex Fourier Series*** $$X[k] = \frac{1}{T}\int_{-T/2}^{T/2}\tilde{x}(t)e^{-j2\pi kt/T}\,dt$$ $$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} X[k]e^{j2\pi kt/T}$$	***Fourier Transform*** $$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$ $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$$	***Non-periodic***
	Discrete	***Continuous***	***Freq. Domain***

Table 3: Selected Fourier, Laplace and z-transform pairs.

Signal	$\longleftrightarrow$	Transform	ROC
$\tilde{x}[n] = \sum_{p=-\infty}^{\infty} \delta[n-pN]$	$\xleftrightarrow{DFT}$	$\tilde{X}[k] = \frac{1}{N}$	
$x[n] = \delta[n]$	$\xleftrightarrow{DTFT}$	$X(e^{j\omega}) = 1$	
$\tilde{x}(t) = \sum_{p=-\infty}^{\infty} \delta(t-pT)$	$\xleftrightarrow{FS}$	$X[k] = \frac{1}{T}$	
$\delta_T[t] = \sum_{p=-\infty}^{\infty} \delta(t-pT)$	$\xleftrightarrow{FT}$	$X(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$	
$\cos(\omega_0 t)$	$\xleftrightarrow{FT}$	$X(j\omega) = \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)$	
$\sin(\omega_0 t)$	$\xleftrightarrow{FT}$	$X(j\omega) = j\pi\delta(\omega+\omega_0) - j\pi\delta(\omega-\omega_0)$	
$x(t) = \begin{cases} 1 & \text{when } \lvert t\rvert \leqslant T_0, \\ 0 & \text{otherwise.} \end{cases}$	$\xleftrightarrow{FT}$	$X(j\omega) = \frac{2\sin(\omega T_0)}{\omega}$	
$x(t) = \frac{1}{\pi t}\sin(\omega_c t)$	$\xleftrightarrow{FT}$	$X(j\omega) = \begin{cases} 1 & \text{when } \lvert\omega\rvert \leqslant \lvert\omega_c\rvert, \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\xleftrightarrow{FT}$	$X(j\omega) = 1$	
$x(t) = \delta(t-t_0)$	$\xleftrightarrow{FT}$	$X(j\omega) = e^{-j\omega t_0}$	
$x(t) = u(t)$	$\xleftrightarrow{FT}$	$X(j\omega) = \pi\delta(w) + \frac{1}{jw}$	
$x[n] = \frac{\omega_c}{\pi}\,\mathrm{sinc}\,\omega_c n$	$\xleftrightarrow{DTFT}$	$X(e^{j\omega}) = \begin{cases} 1 & \text{when } \lvert\omega\rvert < \lvert\omega_c\rvert, \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = 1$	all s
(unit step) $x(t) = u(t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{1}{s}$	
(unit ramp) $x(t) = t$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{1}{s^2}$	
$x(t) = \sin(s_0 t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{s_0}{(s^2 + {s_0}^2)}$	
$x(t) = \cos(s_0 t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{s}{(s^2 + {s_0}^2)}$	
$x(t) = e^{s_0 t}u(t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{1}{s - s_0}$	$\mathfrak{Re}\{s\} > \mathfrak{Re}\{s_0\}$
$x[n] = \delta[n]$	$\xleftrightarrow{z}$	$X(z) = 1$	all z
$x[n] = \delta[n-m]$	$\xleftrightarrow{z}$	$X(z) = z^{-m}$	
$x[n] = u[n]$	$\xleftrightarrow{z}$	$X(z) = \frac{z}{z-1}$	
$x[n] = z_0^n u[n]$	$\xleftrightarrow{z}$	$X(z) = \frac{1}{1 - z_0 z^{-1}}$	$\lvert z\rvert > \lvert z_0\rvert$
$x[n] = -z_0^n u[-n-1]$	$\xleftrightarrow{z}$	$X(z) = \frac{1}{1 - z_0 z^{-1}}$	$\lvert z\rvert < \lvert z_0\rvert$
$x[n] = a^n u[n]$	$\xleftrightarrow{z}$	$X(z) = \frac{z}{z-a}$	$\lvert z\rvert < \lvert a\rvert$

Table 4: Properties of the Discrete-time Fourier Transform.

Property	Time domain	Frequency domain
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\omega}) + bX_2(e^{j\omega})$
Differentiation (frequency)	$nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
Time-shift	$x[n-n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
Frequency-shift	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
Convolution	$x_1[n] * x_2[n]$	$X_1(e^{j\omega})X_2(e^{j\omega})$
Modulation	$x_1[n]x_2[n]$	$\frac{1}{2\pi}X_1(e^{j\omega}) \circledast X_2(e^{j\omega})$
Time-reversal	$x[-n]$	$X(e^{-j\omega})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(e^{j\omega}) = X^*(e^{-j\omega})$
Symmetry (imag)	$\mathfrak{Re}\{x[n]\} = 0$	$X(e^{j\omega}) = -X^*(e^{-j\omega})$
Parseval	$\displaystyle\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left\lvert X(e^{j\omega})\right\rvert^2 d\omega$	

Table 5: Properties of the Fourier series.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1[k] + bX_2[k]$
Differentiation (time)	$\dfrac{d\tilde{x}(t)}{dt}$	$\dfrac{j2\pi k}{T}X[k]$
Time-shift	$\tilde{x}(t-t_0)$	$e^{-j2\pi k t_0/T}X[k]$
Frequency-shift	$e^{j2\pi k_0 t/T}\tilde{x}(t)$	$X[k-k_0]$
Convolution	$\tilde{x}_1(t) \circledast \tilde{x}_2(t)$	$TX_1[k]X_2[k]$
Modulation	$\tilde{x}_1(t)\tilde{x}_2(t)$	$X_1[k] * X_2[k]$
Time-reversal	$\tilde{x}(-t)$	$X[-k]$
Conjugation	$\tilde{x}^*(t)$	$X^*[-k]$
Symmetry (real)	$\mathfrak{Im}\{\tilde{x}(t)\} = 0$	$X[k] = X^*[-k]$
Symmetry (imag)	$\mathfrak{Re}\{\tilde{x}(t)\} = 0$	$X[k] = -X^*[-k]$
Parseval	$\displaystyle\frac{1}{T}\int_{-T/2}^{T/2} \lvert\tilde{x}(t)\rvert^2 dt = \sum_{k=-\infty}^{\infty} \lvert X[k]\rvert^2$	

Table 6: Properties of the Fourier transform.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$
Duality	$X(jt)$	$2\pi x(-\omega)$
Differentiation	$\frac{dx(t)}{dt}$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{1}{j\omega}X(j\omega) + \pi X(j0)\delta(\omega)$
Time-shift	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency-shift	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
Convolution	$x_1(t) * x_2(t)$	$X_1(j\omega)X_2(j\omega)$
Modulation	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(j\omega) * X_2(j\omega)$
Time-reversal	$x(-t)$	$X(-j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Symmetry (real)	$\mathfrak{Im}\{x(t)\} = 0$	$X(j\omega) = X^*(-j\omega)$
Symmetry (imag)	$\mathfrak{Re}\{x(t)\} = 0$	$X(j\omega) = -X^*(-j\omega)$
Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
Parseval	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\, d\omega$	

Table 7: Properties of the z-transform.

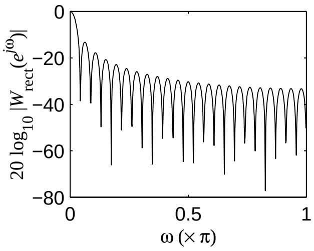
Property	Time domain	z-domain	ROC
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Time-shift	$x[n - n_0]$	$z^{-n_0}X(z)$	$R_x^\dagger$
Scaling in z	$z_0^n x[n]$	$X(z/z_0)$	$\lvert z_0\rvert R_x$
Differentiation in z	$nx[n]$	$-z\frac{dX(z)}{dz}$	$R_x^\dagger$
Time-reversal	$x[-n]$	$X(1/z)$	$1/R_x$
Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(z) = X^*(z^*)$	
Symmetry (imag)	$\mathfrak{Re}\{X[n]\} = 0$	$X(z) = -X^*(z^*)$	
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Initial value	$x[n] = 0, n < 0 \Rightarrow x[0] = \lim_{z\to\infty} X(z)$		

† $z = 0$ or $z = \infty$ may have been added or removed from the ROC.

Table 8: Commonly used window functions.

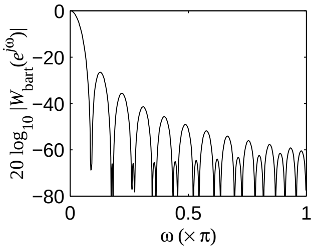
Rectangular:

$$w_{\text{rect}}[n] = \begin{cases} 1 & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



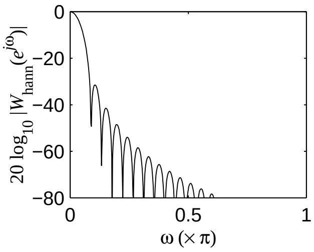
Bartlett (triangular):

$$w_{\text{bart}}[n] = \begin{cases} 2n/M & \text{when } 0 \leqslant n \leqslant M/2, \\ 2 - 2n/M & \text{when } M/2 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



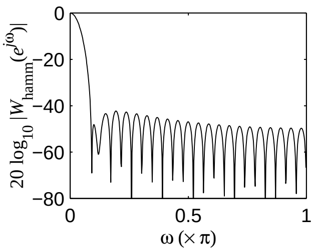
Hanning:

$$w_{\text{hann}}[n] = \begin{cases} \frac{1}{2} - \frac{1}{2}\cos(2\pi n/M) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



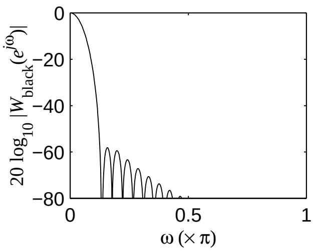
Hamming:

$$w_{\text{hamm}}[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



Blackman:

$$w_{\text{black}}[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) \\ \quad + 0.08\cos(4\pi n/M) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



Type of Window	Peak Side-Lobe Amplitude (Relative; dB)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)
Rectangular	-13	$4\pi/(M+1)$	-21
Bartlett	-25	$8\pi/M$	-25
Hanning	-31	$8\pi/M$	-44
Hamming	-41	$8\pi/M$	-53
Blackman	-57	$12\pi/M$	-74