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Digital Control Systems: Shaping the Dynamic Response + Some Applications

ELEC 3004: **Systems**: Signals & Controls
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Lecture 22

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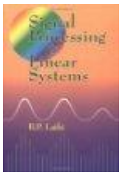
May 25, 2017

Lecture Schedule:

Week	Date	Lecture Title
1	28-Feb	Introduction
	2-Mar	Systems Overview
2	7-Mar	Systems as Maps & Signals as Vectors
	9-Mar	Systems: Linear Differential Systems
3	14-Mar	Sampling Theory & Data Acquisition
	16-Mar	Aliasing & Antialiasing
4	21-Mar	Discrete Time Analysis & Z-Transform
	23-Mar	Second Order LTID (& Convolution Review)
5	28-Mar	Frequency Response
	30-Mar	Filter Analysis
6	4-Apr	Digital Filters (IIR) & Filter Analysis
	6-Apr	Digital Filter (FIR)
7	11-Apr	Digital Windows
	13-Apr	FFT
	18-Apr	**Holiday**
	20-Apr	
	25-Apr	
8	27-Apr	Active Filters & Estimation
9	2-May	Introduction to Feedback Control
	4-May	Servoregulation/PID
10	9-May	PID & State-Space
	11-May	State-Space Control
11	16-May	Digital Control Design
	18-May	Stability
12	23-May	State Space Control System Design
	25-May	Shaping the Dynamic Response
13	30-May	System Identification & Information Theory
	1-Jun	Summary and Course Review

Exam announcement

Follow Along Reading:



B. P. Lathi
Signal processing and linear systems
1998
TK5102.9.L38 1998

G. Franklin,
J. Powell,
M. Workman
Digital Control of Dynamic Systems
1990

TJ216.F72 1990
[Available as UQ Ebook]

Today

➔ **State-space** 🡐 **[A stately idea! ☺]**

- FPW
  - Chapter 6: Design of Digital Control Systems Using State-Space Methods
- Friedland
  - Chapter 6: **Shaping The Dynamic Response**

- Lathi Ch. 13
  - § 13.2 Systematic Procedure for Determining State Equations
  - § 13.3 Solution of State Equations

Next Time

More Online Reading Materials


CHAPTER SIX
SHAPING THE DYNAMIC RESPONSE

6.1 INTRODUCTION


CHAPTER THREE
DYNAMICS OF LINEAR SYSTEMS

3.1 DIFFERENTIAL EQUATIONS REVISITED

- Friedland, Control System Design Ch. 6 and 3

➔ http://robotics.itee.uq.edu.au/~elec3004/tutes.html

Shaping the Dynamic Response: Pole Placement (FPW Chapter 6)

Pole Placement (Following FPW – Chapter 6)

- FPW has a slightly different notation:

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}u,$$
$$y = \mathbf{H}\mathbf{x}.$$

$$\mathbf{x}(k+1) = \mathbf{\Phi}\mathbf{x}(k) + \mathbf{\Gamma}u(k),$$
$$y(k) = \mathbf{H}\mathbf{x}(k),$$

$$\mathbf{\Phi} = e^{\mathbf{F}T},$$

$$\mathbf{\Gamma} = \int_0^T e^{\mathbf{F}\eta} d\eta \mathbf{G},$$

Pole Placement

- Start with a simple feedback control law ("controller")

$$u = -\mathbf{K}\mathbf{x} = -[K_1 K_2 \ldots] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}$$

- It's actually a regulator
  ∵ it does not allow for a reference input to the system.
  (there is no "reference" **r** (r = 0))

- Substitute in the difference equation

$$x(k+1) = \Phi x(k) - \Gamma K x(k)$$

- $\mathcal{Z}$ Transform:

$$(zI - \Phi + \Gamma K)X(z) = 0$$

➔Characteristic Eqn:

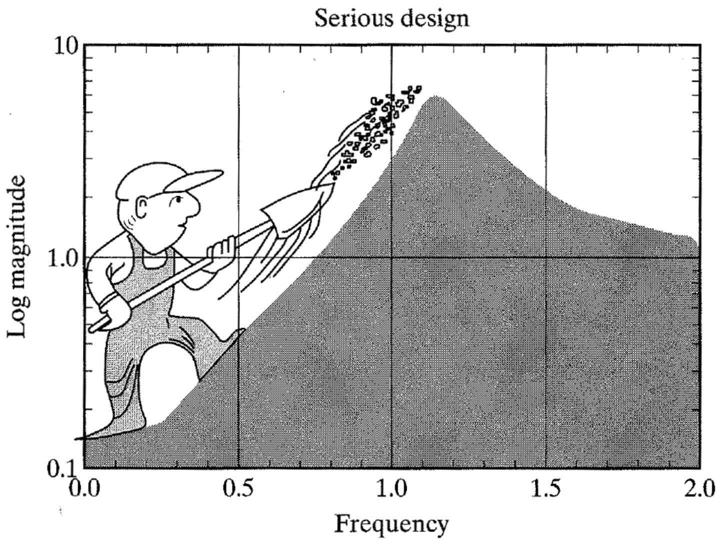
$$\det|zI - \Phi + \Gamma K| = 0$$

Pole Placement

Pole placement: Big idea:

- Arbitrarily select the desired root locations of the closed-loop system and see if the approach will work.
- AKA: full state feedback
  ∵ enough parameters to influence all the closed-loop poles
- Finding the elements of K so that the roots are in the desired locations. Unlike classical design, where we iterated on parameters in the compensator (hoping) to find acceptable root locations, the full state feedback, pole-placement approach guarantees success and allows us to arbitrarily pick any root locations, providing that *n* roots are specified for an n^{th}-order system.

Meaning…



Back to Pole Placement

- Given:

$$z_i = \beta_1, \beta_2, \beta_3, \ldots$$

- This gives the desired control-characteristic equation as:

$$a_c(z) = (z - \beta_1)(z - \beta_2)(z - \beta_3) \ldots =$$

- Now we "just solve" for **K** and "bingo"

Pole Placement Example (FPW p. 241)

Example 6.1: Suppose we want to design a control law for the satellite attitude-control system described by (2.45) with $\boldsymbol{x} = [\boldsymbol{x}_1 \ \ \boldsymbol{x}_2]$. Example 2.13 showed that the discrete model for this system is

$$\boldsymbol{\Phi} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Gamma} = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}.$$

We want to pick z-plane roots of the closed-loop characteristic equation so that the equivalent s-plane roots have a damping ratio of $\zeta = 0.5$ and real part of $s = -1.8$ rad/sec (i.e., $s = -1.8 \pm j3.12$ rad/sec). Using $z = e^{sT}$ with a sample period of $T = 0.1$ sec, we find that $z = 0.8 \pm j0.25$, as shown in Fig. 6.1. The desired characteristic equation is then

$$z^2 - 1.6z + 0.70 = 0, \tag{6.9}$$

and the evaluation of (6.7) for any control law $\mathbf{K}$ leads to

$$\det \left| z \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} [K_1 \quad K_2] \right| = 0$$

or

$$z^2 + (TK_2 + (T^2/2)K_1 - 2)z + (T^2/2)K_1 - TK_2 + 1 = 0. \tag{6.10}$$

Pole Placement Example (FPW p. 241)

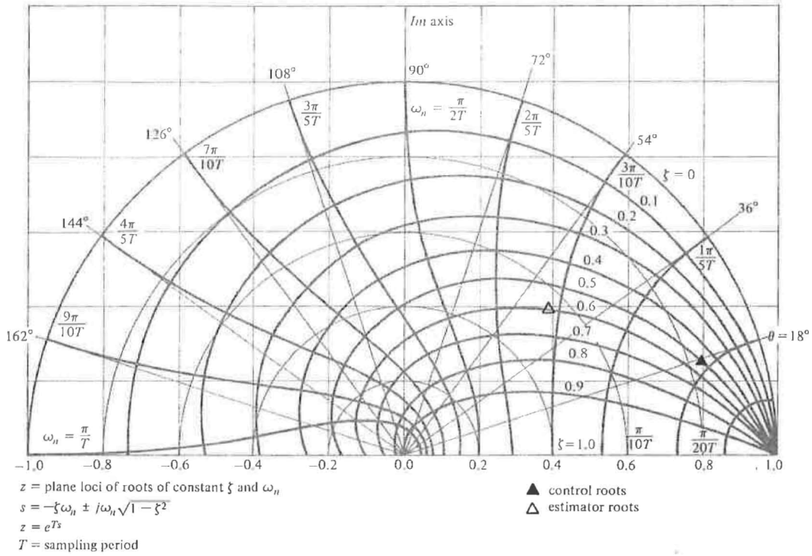
Equating coefficients in (6.9) and (6.10) with like powers of z, we obtain two simultaneous equations in the two unknown elements of $\mathbf{K}$:

$$TK_2 + (T^2/2)K_1 - 2 = -1.6,$$
$$(T^2/2)K_1 - TK_2 + 1 = 0.70,$$

which are easily solved for the coefficients and evaluated for $T = 0.1$ sec:

$$K_1 = \frac{0.10}{T^2} = 10, \qquad K_2 = \frac{0.35}{T} = 3.5.$$

Pole Placement Example (FPW p. 241)



Ackermann's Formula (FPW p. 245)

- Gains maybe approximated with:

$$\boxed{\mathbf{K} = [0 \ldots 0 \quad 1][\mathbf{\Gamma} \quad \mathbf{\Phi\Gamma} \quad \mathbf{\Phi}^2\mathbf{\Gamma} \ldots \mathbf{\Phi}^{n-1}\mathbf{\Gamma}]^{-1}\alpha_c(\mathbf{\Phi}),}$$

- Where: C = controllability matrix, *n* is the order of the system (or number of state elements) and α_c:

$$\mathcal{C} = [\mathbf{\Gamma} \quad \mathbf{\Phi\Gamma} \ldots]$$

$$\alpha_c(\mathbf{\Phi}) = \mathbf{\Phi}^n + \alpha_1\mathbf{\Phi}^{n-1} + \alpha_2\mathbf{\Phi}^{n-2} + \cdots + \alpha_n\mathbf{I},$$

  - α_i: coefficients of the desired characteristic equation

$$\alpha_c(z) = |\, z\mathbf{I} - \mathbf{\Phi} + \mathbf{\Gamma K} \,| = z^n + \alpha_1 z^{n-1} + \cdots + \alpha_n.$$

Ackermann's Formula Example (FPW p.246)

Example 6.2: Applying Ackermann's formula to the satellite attitude-control system of Example 6.1, we find from (6.9) that

$$\alpha_1 = -1.6, \qquad \alpha_2 = +0.70,$$

and therefore

$$\alpha_c(\Phi) = \begin{bmatrix} 1 & 2T \\ 0 & 1 \end{bmatrix} - 1.6 \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} + 0.70 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.4T \\ 0 & 0.1 \end{bmatrix}.$$

Furthermore, we find that

$$[\boldsymbol{\Gamma} \;\; \boldsymbol{\Phi\Gamma}] = \begin{bmatrix} T^2/2 & 3T^2/2 \\ T & T \end{bmatrix}$$

and

$$[\boldsymbol{\Gamma} \;\; \boldsymbol{\Phi\Gamma}]^{-1} = 1/T^2 \begin{bmatrix} -1 & +3T/2 \\ 1 & -T/2 \end{bmatrix},$$

and finally

$$\mathbf{K} = [K_1 \;\; K_2] = (1/T^2)[0 \;\; 1] \begin{bmatrix} -1 & 3T/2 \\ 1 & -T/2 \end{bmatrix} \begin{bmatrix} 0.1 & 0.4T \\ 0 & 0.1 \end{bmatrix};$$

therefore

$$[K_1 \;\; K_2] = \frac{1}{T^2}[0.1 \;\; 0.35T]$$
$$= [10 \quad 3.5],$$

which is the same result as that obtained earlier.

Shaping the Dynamic Response: SISO (Friedland Chapter 6)

SDR: Introduction

6.1 INTRODUCTION

At last we have arrived at the point of using state-space methods for control system design. In this chapter we will develop a simple method of designing a control system for a process in which all the state variables are accessible for measurement—the method known as *pole-placement*. We will find that in a controllable system, with all the state variables accessible for measurement, it is possible to place the closed-loop poles anywhere we wish in the complex s plane. This means that we can, in principle, completely specify the closed-loop dynamic performance of the system. In principle, we can start with a sluggish open-loop system and force it to behave with alacrity; in principle, we can start with a system that has very little open-loop damping and provide any amount of damping desired. Unfortunately, however, what can be attained in principle may not be attainable in practice. Speeding the response of a sluggish system requires the use of large control signals which the actuator (or power supply) may not be capable of delivering. The consequence is generally that the actuator saturates at the largest signal that it can supply. In some instances the system behavior may be acceptable in spite of the saturation. But in other cases the effect of saturation is to make the closed-loop system unstable. It is usually not possible to alter open-loop dynamic behavior very drastically without creating practical difficulties.

Adding a great deal of damping to a system having poles near the imaginary axis is also problematic, not only because of the magnitude of the control signals needed, but also because the control system gains are very sensitive to the location of the open-loop poles. Slight changes in the open-loop pole

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SDR: Introduction [2]

location may cause the closed-loop system behavior to be very different from that for which it is designed.

We will first address the design of a regulator. Here the problem is to determine the gain matrix G in a linear feedback law

$$u = -Gx \tag{6.1}$$

which shapes the dynamic response of the process in the absence of disturbances and reference inputs. Afterward we shall consider the more general problem of determining the matrices G and G_0 in the linear control law

$$u = -Gx - G_0x_0 \tag{6.2}$$

where x_0 is the vector of exogenous variables. The reason it is necessary to separate the exogenous variables from the process state x, rather than deal directly with the metastate

$$\mathbf{x} = \begin{bmatrix} x \\ \hline x_0 \end{bmatrix} \tag{6.3}$$

introduced in Chap. 5, is that in developing the theory for the design of the gain matrix, we must assume that the underlying process is *controllable*. Since the exogenous variables are not true state variables, but additional inputs that cannot be affected by the control action, they cannot be included in the state vector when using a design method that requires controllability.

SDR: Introduction [3]

The assumption that all the state variables are accessible to measurement in the regulator means that the gain matrix G in (6.1) is permitted to be any function of the state x that the design method requires. In most practical instances, however, the state variables are not all accessible for measurement. The feedback control system design for such a process must be designed to use only the measurable output of the process

$$y = Cx$$

where y is a vector of lower dimension than x. In some cases it may be possible to determine the gain matrix G_y for a control law of the form

$$u = -G_y y \tag{6.4}$$

which produces acceptable performance. But more often it is not possible to do so. It is then necessary to use a more general feedback law, of the form

$$u = -G\hat{x} \tag{6.5}$$

where $\hat{x}$ is the state of an appropriate dynamic system known as an "observer." The design of observers is the subject of Chap. 7. And in Chap. 8, we shall show that when a feedback law of the form of (6.5) is used with a properly designed observer, the dynamic properties of the overall system can be specified at will, subject to practical limitations on control magnitude and accuracy of implementation.

Design of regulators for single-input, single-output systems

6.2 DESIGN OF REGULATORS FOR SINGLE-INPUT, SINGLE-OUTPUT SYSTEMS

The present section is concerned with the design of a gain matrix

$$G = g' = [g_1, g_2, \ldots, g_k] \tag{6.6}$$

for the single-input, single-output system

$$\dot{x} = Ax + Bu \tag{6.7}$$

where

$$B = b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix} \tag{6.8}$$

With the control law $u = -Gx = -g'x$ (6.7) becomes

$$\dot{x} = (A - bg')x$$

Our objective is to find the matrix $G = g'$ which places the poles of the closed-loop dynamics matrix

$$A_c = A - bg' \tag{6.9}$$

Design of regulators for single-input, single-output systems

at the locations desired. We note that there are k gains $g_1, g_2, \ldots, g_k$ and k poles for a kth order system, so there are precisely as many gains as needed to specify each of the closed-loop poles.

One way of determining the gains would be to set up the characteristic polynomial for A_c:

$$|sI - A_c| = |sI - A + bg'| = s^k + \bar{a}_1 s^{k-1} + \cdots + \bar{a}_k \tag{6.10}$$

The coefficients $\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_k$ of the powers of s in the characteristic polynomial will be functions of the k unknown gains. Equating these functions to the numerical values desired for $\bar{a}_1, \ldots, \bar{a}_k$ will result in k simultaneous equations the solution of which will yield the desired gains $g_1, \ldots, g_k$.

This is a perfectly valid method of determining the gain matrix g', but it entails a substantial amount of calculation when the order k of the system is higher than 3 or 4. For this reason, we would like to develop a direct formula for g in terms of the coefficients of the open-loop and closed-loop characteristic equations.

If the original system is in the companion form given in (3.90), the task is particularly easy, because

$$A = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{k-1} & -a_k \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \tag{6.11}$$

Design of regulators for single-input, single-output systems

$$bg' = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} [g_1, g_2, \ldots, g_k] = \begin{bmatrix} g_1 & g_2 & \cdots & g_k \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

Hence

$$A_c = A - bg' = \begin{bmatrix} -a_1 - g_1 & -a_2 - g_2 & \cdots & -a_k - g_k \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

The gains $g_1, \ldots, g_k$ are simply added to the coefficients of the open-loop A matrix to give the closed-loop matrix A_c. This is also evident from the block-diagram representation of the closed-loop system as shown in Fig. 6.1. Thus for a system in the companion form of Fig. 6.1, the gain matrix elements are given by

$$a_i + g_i = \hat{a}_i \qquad i = 1, 2, \ldots, k$$

or

$$g = \hat{a} - a \tag{6.12}$$

where

$$a = \begin{bmatrix} a_1 \\ \vdots \\ a_k \end{bmatrix} \qquad \hat{a} = \begin{bmatrix} \hat{a}_1 \\ \vdots \\ \hat{a}_k \end{bmatrix} \tag{6.13}$$

Design of regulators for single-input, single-output systems

are vectors formed from the coefficients of the open-loop and closed-loop characteristic equations, respectively.

The dynamics of a typical system are usually not in companion form. It is necessary to transform such a system into companion form before (6.12) can be used. Suppose that the state of the transformed system is $\bar{x}$, achieved through the transformation

$$\bar{x} = Tx \tag{6.14}$$

Then, as shown in Chap. 3,

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{b}u \tag{6.15}$$

where

$$\bar{A} = TAT^{-1} \quad \text{and} \quad \bar{b} = Tb$$

For the transformed system the gain matrix is

$$\bar{g} = \hat{a} - \bar{a} = \hat{a} - a \tag{6.16}$$

since $\bar{a} = a$ (the characteristic equation being invariant under a change of state variables). The desired control law in the original system is

$$u = -g'x = -g'T^{-1}\bar{x} = -\bar{g}'\bar{x} \tag{6.17}$$

From (6.17) we see that

$$\bar{g}' = g'T^{-1}$$

Thus the gain in the original system is

$$g = T'\bar{g} = T'(\hat{a} - a) \tag{6.18}$$

Design of regulators for single-input, single-output systems

In words, the desired gain matrix for a general system is the difference between the coefficient vectors of the desired and actual characteristic equation, premultiplied by the inverse of the transpose of the matrix T that transforms the general system into the companion form of (3.90), the A matrix of which has the form (6.11).

The desired matrix T is obtained as the product of two matrices U and V:

$$T = VU \tag{6.19}$$

The first of these matrices transforms the original system into an intermediate system

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} \tag{6.20}$$

in the second companion form (3.107) and the second transformation U transforms the intermediate system into the first companion form.

Consider the intermediate system

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{b}u \tag{6.21}$$

with $\tilde{A}$ and $\tilde{b}$ in the form of (3.107). Then we must have

$$\tilde{A} = UAU^{-1} \quad \text{and} \quad \tilde{b} = Ub \tag{6.22}$$

Design of regulators for single-input, single-output systems

The desired matrix U is precisely the inverse of the controllability test matrix Q of Sec. 5.4. To prove this fact, we must show that

$$U^{-1}\tilde{A} = AU^{-1} \tag{6.23}$$

or

$$Q\tilde{A} = AQ \tag{6.24}$$

Now, for a single-input system

$$Q = [b, Ab, \ldots, A^{k-1}b]$$

Thus, with $\tilde{A}$ given by (3.107), the left-hand side of (6.23) is

$$Q\tilde{A} = [b, Ab, \ldots, A^{k-1}b]\begin{bmatrix} 0 & 0 & \cdots & -a_k \\ 1 & 0 & \cdots & -a_{k-1} \\ 0 & 1 & \cdots & -a_{k-2} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -a_1 \end{bmatrix}$$

$$= [Ab, A^2b, \ldots, A^{k-1}b, -a_k b - a_{k-1}Ab - \cdots - a_k A^{k-1}b] \tag{6.25}$$

The last term in (6.25) is

$$(-a_k I - a_{k-1}A - \cdots - a_k A^{k-1})b \tag{6.26}$$

Now, by the Cayley-Hamilton theorem, (see Appendix):

$$A^k = -a_1 A^{k-1} - a_2 A^{k-2} - \cdots - a_k I$$

so (6.26) is $A^k b$. Thus the left-hand side of (6.24) as given by (6.25) is

$$Q\tilde{A} = [Ab, A^2b, \ldots, A^k b] = A[b, Ab, \ldots, A^{k-1}b] = AQ$$

which is the desired result.

Ex: Servo Motor Control

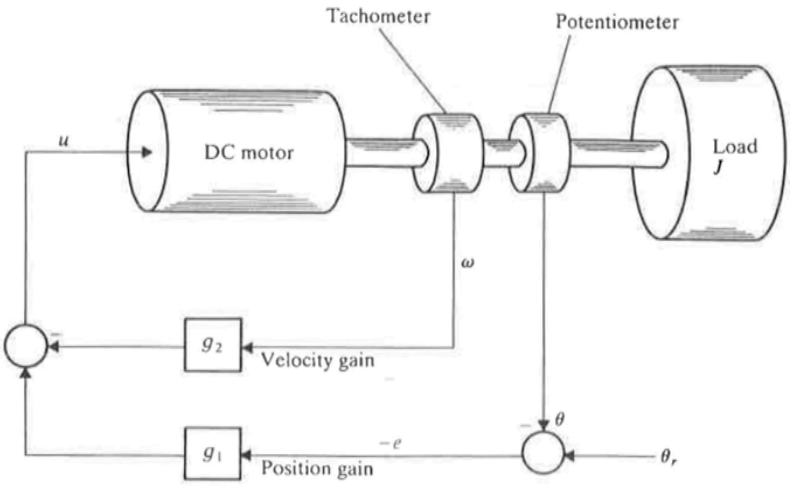

Figure 6.2 Implementation of an instrument servo.

Ex: Servo Motor Control [2]

Example 6A Instrument servo A dc motor driving an inertial load constitutes a simple instrument servo for keeping the load at a fixed position.

As shown in Chap. 2 (Example 2B), the state-space equations for the motor-driven inertia are

$$\dot{\theta} = \omega \tag{6A.1}$$

$$\dot{\omega} = -\alpha\omega + \beta u \tag{6A.2}$$

where θ is the angular position of the load, ω is the angular velocity, u is the applied voltage, and α and β are constants that depend on the physical parameters of the motor and load:

$$\alpha = -K^2/JR \qquad \beta = K/JR$$

If the desired position θ_r is a constant then we can define the servo error

$$e = \theta - \theta_r$$

Then

$$\dot{e} = \dot{\theta} - \dot{\theta}_r = \omega \qquad (\theta_r = \text{const}) \tag{6A.3}$$

and (6A.3) replaces (6A.1) to give

$$\begin{bmatrix} \dot{e} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\alpha \end{bmatrix} \begin{bmatrix} e \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u \tag{6A.4}$$

The angular position measurement can be instrumented by a potentiometer on the motor shaft and the angular velocity by a tachometer. Thus, the closed-loop system would have the configuration illustrated in Fig. 6.2. Note that the position gain is shown multiplying the negative of the system error which in turn is added to the control signal. This is consistent with the convention normally used for servos, wherein the position gain multiplies the difference $\theta_r - \theta$ between the reference and the actual positions. The quantity e defined above (6A.3) is the negative of the system error as normally defined in elementary texts.

The characteristic polynomial of the system is

$$|sI - A| = \begin{vmatrix} s & -1 \\ 0 & s + \alpha \end{vmatrix} = s^2 + \alpha s$$

Thus

$$a = \begin{bmatrix} \alpha \\ 0 \end{bmatrix}$$

The controllability test matrix Q and the matrix W are given respectively by

$$Q = [b, Ab] = \begin{bmatrix} 0 & \beta \\ \beta & -\alpha\beta \end{bmatrix} \qquad W = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$$

Ex: Servo Motor Control [3]

Thus

$$QW = \begin{bmatrix} 0 & \beta \\ \beta & 0 \end{bmatrix} = (QW)'$$

and

$$[(QW)']^{-1} = \begin{bmatrix} 0 & 1/\beta \\ 1/\beta & 0 \end{bmatrix}$$

Thus the desired gain matrix, by the Bass-Gura formula (6.34), is

$$g = \begin{bmatrix} 0 & 1/\beta \\ 1/\beta & 0 \end{bmatrix} \begin{bmatrix} \bar{a}_1 - \alpha \\ \bar{a}_2 \end{bmatrix} = \begin{bmatrix} \bar{a}_2/\beta \\ (\bar{a}_1 - \alpha)/\beta \end{bmatrix} \tag{6A.5}$$

where $\bar{a}_1$ and $\bar{a}_2$ are the coefficients of the desired characteristic polynomial.

While the above calculation illustrates the general procedure, the gains could have been more easily computed directly. For a control law of the form

$$u = -g_1 e - g_2 \omega$$

(6A.4) becomes

$$\dot{e} = \omega$$

$$\dot{\omega} = -g_1 \beta e - (\alpha + \beta g_2)\omega$$

which has the closed-loop matrix

$$A_c = \begin{bmatrix} 0 & 1 \\ -g_1\beta & -(\alpha + g_2\beta) \end{bmatrix}$$

with the characteristic equation

$$|sI - A_c| = s^2 + (\alpha + g_2\beta)s + g_1\beta$$

Thus

$$\bar{a}_1 = \alpha + g_2\beta \qquad \bar{a}_2 = g_1\beta$$

Ex: Servo Motor Control [4]

or

$$g_1 = \bar{a}_2/\beta \qquad g_2 = (\bar{a}_1 - \alpha)/\beta$$

which is the same as (6A.5).

Note that the position and velocity gains g_1 and g_2, respectively, are proportional to the amounts we wish to move the coefficients from their open-loop positions. The position gain g_1 is necessary to produce a stable system: $\bar{a}_2 > 0$. But if the designer is willing to settle for $a_1 = \alpha$, i.e., to accept the open-loop damping, then the gain g_2 can be zero. This of course eliminates the need for a tachometer and reduces the hardware cost of the system. It is also possible to alter the system damping without the use of a tachometer, by using an estimate $\hat{\omega}$ of the angular velocity ω. This estimate is obtained by means of an observer as discussed in Chap. 7.

Break ☺

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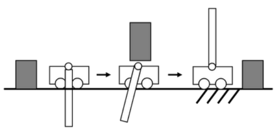
</assessable>

- Nothing beyond this point is on the exam.
  (except for the exam review ☺)
- Do not pay attention.
- Do not attempt to learn.

Application Example 1: Inverted Pendulum

Digital Control

Wikipedia,
Cart and pole



$$L = \frac{1}{2}Mv_1^2 + \frac{1}{2}mv_2^2 - mg\ell\cos\theta$$

where v_1 is the velocity of the cart and v_2 is the velocity of the point mass m. v_1 and v_2 can be expressed in terms of x and θ by writing the velocity as the first derivative of the position;

$$v_1^2 = \dot{x}^2$$
$$v_2^2 = \left(\frac{d}{dt}(x - \ell\sin\theta)\right)^2 + \left(\frac{d}{dt}(\ell\cos\theta)\right)^2$$

Simplifying the expression for v_2 leads to:

$$v_2^2 = \dot{x}^2 - 2\ell\dot{x}\dot{\theta}\cos\theta + \ell^2\dot{\theta}^2$$

The Lagrangian is now given by:

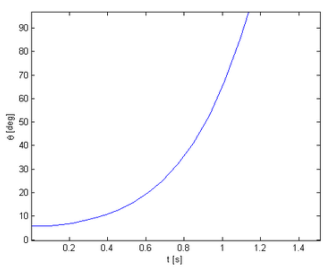
$$L = \frac{1}{2}(M+m)\dot{x}^2 - m\ell\dot{x}\dot{\theta}\cos\theta + \frac{1}{2}m\ell^2\dot{\theta}^2 - mg\ell\cos\theta$$

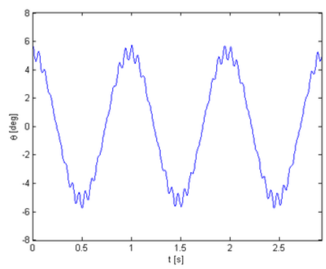
and the equations of motion are:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = F$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0$$

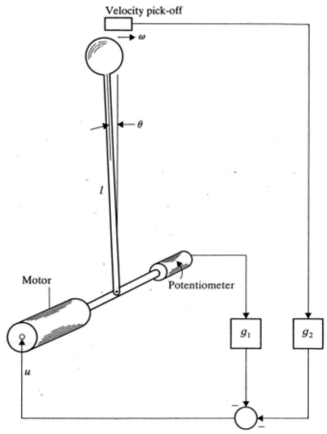
substituting L in these equations and simplifying leads to the equations that describe the motion o

$$(M+m)\ddot{x} - m\ell\ddot{\theta}\cos\theta + m\ell\dot{\theta}^2\sin\theta = F$$
$$\ell\ddot{\theta} - g\sin\theta = \ddot{x}\cos\theta$$





Inverted Pendulum



$$L = \frac{1}{2}Mv_1^2 + \frac{1}{2}mv_2^2 - mg\ell\cos\theta$$

where v_1 is the velocity of the cart and v_2 is the velocity of the point mass m. v_1 and v_2 can be expressed in terms of x and θ by writing the velocity as the first derivative of the position;

$$v_1^2 = \dot{x}^2$$

$$v_2^2 = \left(\frac{d}{dt}(x - \ell\sin\theta)\right)^2 + \left(\frac{d}{dt}(\ell\cos\theta)\right)^2$$

Simplifying the expression for v_2 leads to:

$$v_2^2 = \dot{x}^2 - 2\ell\dot{x}\dot{\theta}\cos\theta + \ell^2\dot{\theta}^2$$

The Lagrangian is now given by:

$$L = \frac{1}{2}(M+m)\dot{x}^2 - m\ell\dot{x}\dot{\theta}\cos\theta + \frac{1}{2}m\ell^2\dot{\theta}^2 - mg\ell\cos\theta$$

and the equations of motion are:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = F$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0$$

substituting L in these equations and simplifying leads to the equations that describe the motion o

$$(M+m)\ddot{x} - m\ell\ddot{\theta}\cos\theta + m\ell\dot{\theta}^2\sin\theta = F$$

$$\ell\ddot{\theta} - g\sin\theta = \ddot{x}\cos\theta$$

Inverted Pendulum – Equations of Motion

- The equations of motion of an inverted pendulum (under a small angle approximation) may be linearized as:

$$\dot{\theta} = \omega$$

$$\dot{\omega} = \ddot{\theta} = Q^2\theta + Pu$$

Where:

$$Q^2 = \left(\frac{M+m}{Ml}\right)g$$

$$P = \frac{1}{Ml}.$$

If we further assume unity ***Ml*** ($Ml \approx 1$), then $P \approx 1$

Inverted Pendulum –State Space

- We then select a state-vector as:

$$\boldsymbol{x} = \begin{bmatrix} \theta \\ \omega \end{bmatrix}, \text{ hence } \dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \omega \\ \dot{\omega} \end{bmatrix}$$

- Hence giving a state-space model as:

$$A = \begin{bmatrix} 0 & 1 \\ Q^2 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- The resolvent of which is:

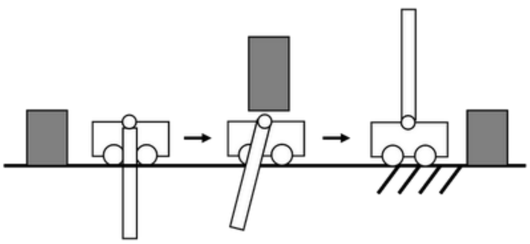
$$\Phi(s) = (sI - A)^{-1} = \begin{bmatrix} s & -1 \\ -Q^2 & s \end{bmatrix}^{-1} = \frac{1}{s^2 - Q^2} \begin{bmatrix} s & 1 \\ Q^2 & s \end{bmatrix}$$

- And a state-transition matrix as:

$$\Phi(t) = \begin{bmatrix} \cosh Qt & \dfrac{\sinh Qt}{Q} \\ Q\ sinh\ Qt & \cosh Qt \end{bmatrix}$$

Cart & Pole in State-Space With Obstacles?

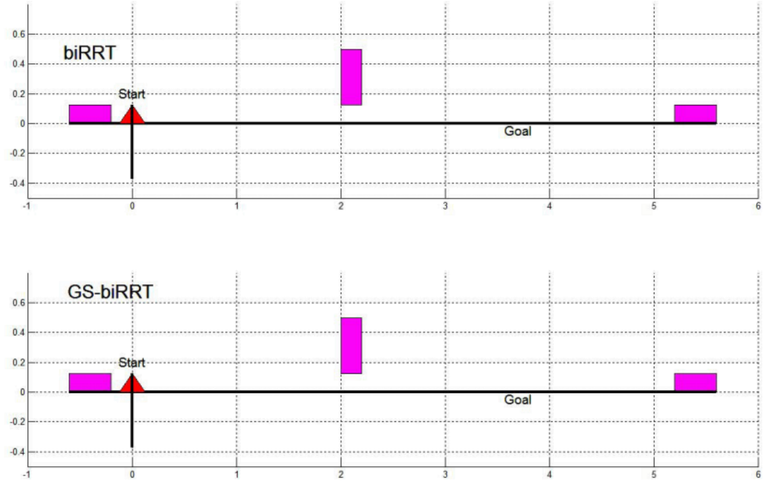
Swing-up is a little more than stabilization…



See also: METR4202 – Tutorial 11:
http://robotics.itee.uq.edu.au/~metr4202/tpl/t11-Week11-pendulum.pdf

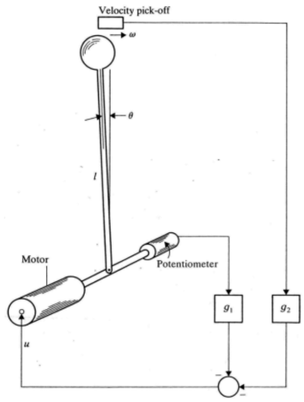
Cart & Pole in State-Space

Swing-up is a little more than stabilization…



Application Example 2: Inverted Pendulum Mark II

Inverted Pendulum (Friedland, Ch. 6 p. 232)



Example 6B Stabilization of an inverted pendulum An inverted pendulum can readily be stabilized by a closed-loop feedback system, just as a person of moderate dexterity can do it.

A possible control system implementation is shown in Fig. 6.3, for a pendulum constrained to rotate about a shaft at its bottom point. The actuator is a dc motor. The angular position of the pendulum, being equal to the position of the shaft to which it is attached, is measured by means of a potentiometer. The angular velocity in this case can be measured by a "velocity pick-off" at the top of the pendulum. Such a device could consist of a coil of wire

Inverted Pendulum [2]

in a magnetic field created by a small permanent magnet in the pendulum bob. The induced voltage in the coil is proportional to the linear velocity of the bob as it passes the coil. And since the bob is at a fixed distance from the pivot point the linear velocity is proportional to the angular velocity. The angular velocity could of course also be measured by means of a tachometer on the dc motor shaft.

As determined in Prob. 2.2, the dynamic equations governing the inverted pendulum in which the point of attachment does not translate is given by

$$\begin{aligned} \dot{\theta} &= \omega \\ \dot{\omega} &= \Omega^2\theta - \alpha\omega + \beta u \end{aligned} \tag{6B.1}$$

where α and β are given in Example 6A, with the inertia J being the total reflected inertia:

$$J = J_m + ml^2$$

where m is the pendulum bob mass and l is the distance of the bob from the pivot. The natural frequency Ω is given by

$$\Omega^2 = \frac{mgl}{J + ml^2} = \frac{g}{l + J/ml}$$

(Note that the motor inertia J_m affects the natural frequency.)

Since the linearization is valid only when the pendulum is nearly vertical, we shall assume that the control objective is to maintain $\theta = 0$. Thus we have a simple regulator problem.

The matrices A and b for this problem are

$$A = \begin{bmatrix} 0 & 1 \\ \Omega^2 & -\alpha \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ \beta \end{bmatrix}$$

Inverted Pendulum [3]

The open-loop characteristic polynomial is

$$|sI - A| = \begin{vmatrix} s & -1 \\ -\Omega^2 & s+\alpha \end{vmatrix} = s^2 + \alpha s - \Omega^2$$

Thus

$$a_1 = \alpha$$
$$a_2 = -\Omega^2$$

The open-loop system is unstable, of course.

The controllability test matrix and the W matrix are given respectively by

$$Q = \begin{bmatrix} 0 & \beta \\ \beta & -\alpha\beta \end{bmatrix} \qquad W = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$$

(which are the same as they were for the instrument servo). And

$$[(QW)']^{-1} = \begin{bmatrix} 0 & 1/\beta \\ 1/\beta & 0 \end{bmatrix}$$

Thus the gain matrix required for pole placement using (6.34), is

$$g = \begin{bmatrix} 0 & 1/\beta \\ 1/\beta & 0 \end{bmatrix} \begin{bmatrix} (\bar{a}_1 - \alpha) \\ \bar{a}_2 + \Omega^2 \end{bmatrix} = \begin{bmatrix} (\bar{a}_2 + \Omega^2)/\beta \\ (\bar{a}_1 - \alpha)/\beta \end{bmatrix}$$

Example 6C Control of spring-coupled masses The dynamics of a pair of spring-coupled masses, shown in Fig. 3.7(a), were shown in Example 3I to have the matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -K/\bar{M} & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Inverted Pendulum [4]

The system has the characteristic polynomial

$$D(s) = s^4 + (K/\bar{M})s^2$$

Hence $\qquad a_1 = a_3 = a_4 = 0, \qquad a_2 = K/\bar{M}.$

The controllability test and W matrices are given, respectively, by

$$Q = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -K/\bar{M} \\ 1 & 0 & -K/\bar{M} & 0 \end{bmatrix} \qquad W = \begin{bmatrix} 1 & 0 & K/\bar{M} & 0 \\ 0 & 1 & 0 & K/\bar{M} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (6C.1)$$

Multiplying we find that

$$QW = (QW)' = (QW)^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad (6C.2)$$

(This rather simple result is not really as surprising as it may at first seem. Note that A is in the first companion form but using the right-to-left numbering convention. If the left-to-right numbering convention were used the A matrix would already be in the companion form of (6.11) and would not require transformation. The transformation matrix T given by (6C.2) has the effect of changing the state variable numbering order from left-to-right to right-to-left, and vice versa.)

The gain matrix g is thus given by

$$g = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 - K/\bar{M} \\ \bar{a}_3 \\ \bar{a}_4 \end{bmatrix} = \begin{bmatrix} \bar{a}_4 \\ \bar{a}_3 \\ \bar{a}_2 - K/\bar{M} \\ \bar{a}_1 \end{bmatrix}$$

A suitable pole "constellation" for the closed-loop process might be a Butterworth pattern as discussed in Sec. 6.5. To achieve this pattern the characteristic polynomial should be of the form

$$\bar{D}(s) = s^4 + (1+\sqrt{3})\Omega s^3 + (2+\sqrt{3})\Omega^2 s^2 + (1+\sqrt{3})\Omega^3 s + \Omega^4$$

Thus

$$\bar{a}_1 = (1+\sqrt{3})\Omega$$

$$\bar{a}_2 = (2+\sqrt{3})\Omega^2$$

$$\bar{a}_3 = (1+\sqrt{3})\Omega^3$$

$$\bar{a}_4 = \Omega^4$$

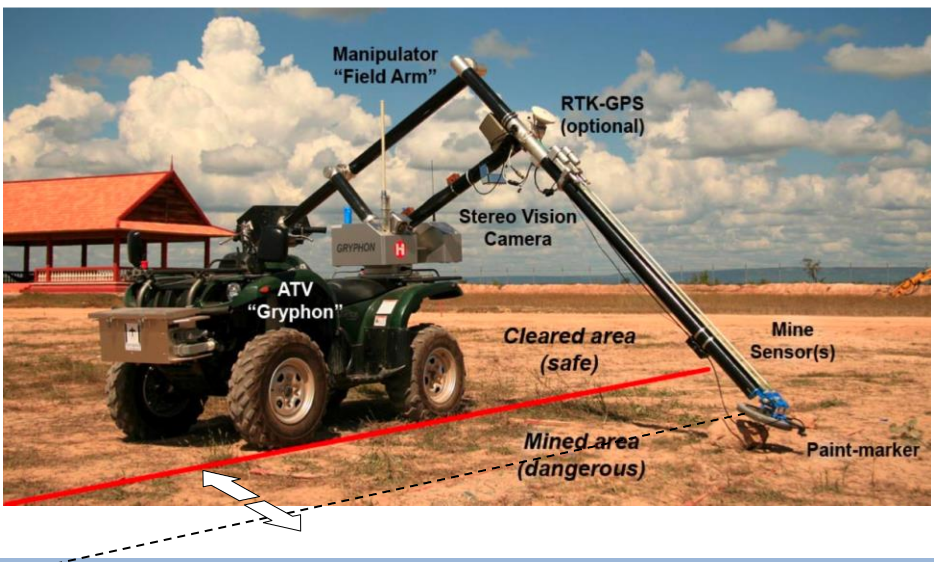
Thus the gain matrix g is given by

$$g = \begin{bmatrix} \Omega^4 \\ (1+\sqrt{3})\Omega^3 \\ (2+\sqrt{3})\Omega^2 - K/\bar{M} \\ (1+\sqrt{3})\Omega \end{bmatrix}$$

Gryphon:

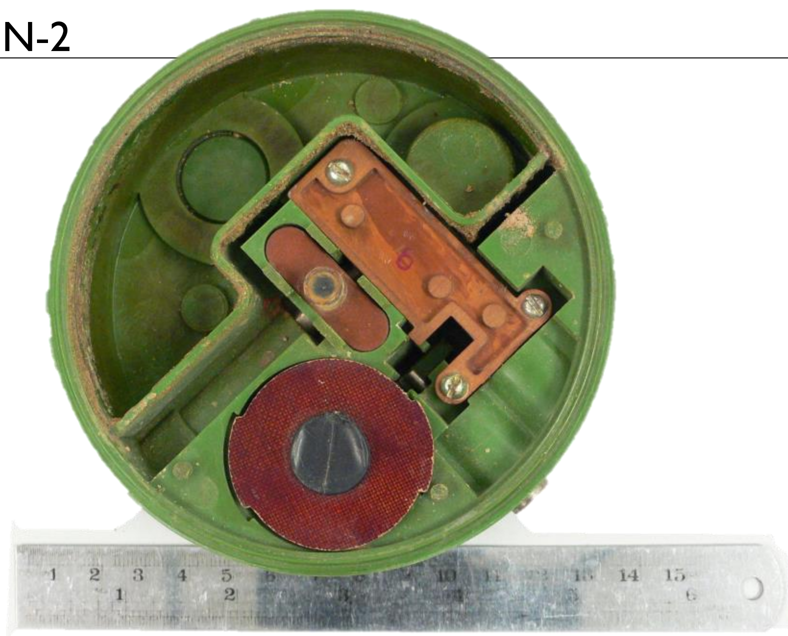
Today: "Bang-Bang Control!"

Gryphon: Mine Scanning Robot



Landmines: Smart for one, dumb for all…

Ex: PMN-2



Land Mines: **Highly Variable**

- Little metal
  ∴ "High-sensitivity" detectors / instruments

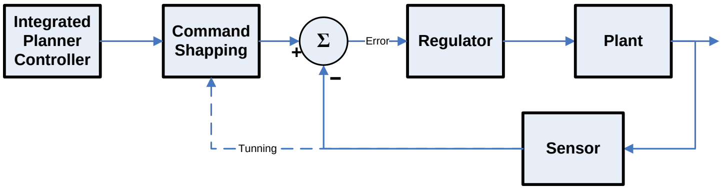
- **Highly Variable** (Example: PMN-2):
  - 3-stage detonation
  - Anti-thwart
  - All mechanical
  - Poor construction detectors / instruments

  ∴ Focus on **manipulating sensor** instead of complex sensing ???,,,

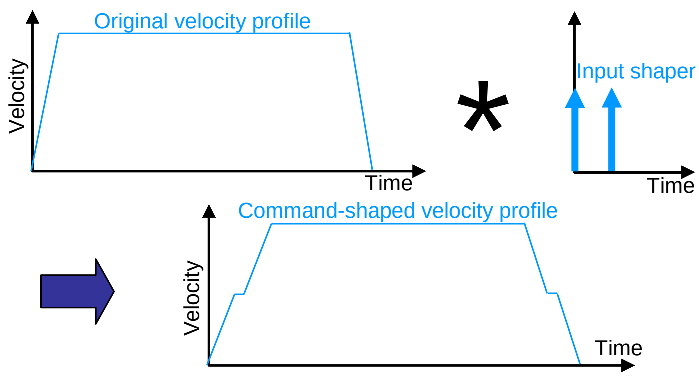
Sensor Mobility Is Critical

Back to Gyrphon …

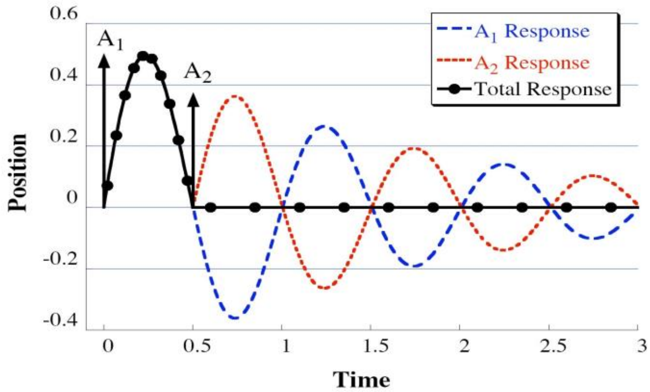
Robust Control:
Command Shaping for Vibration Reduction



Command Shaping



Command Shaping in Position Space



Command Shaping: Zero Vibration and Derivative

$$K = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \qquad i = 1,2$$

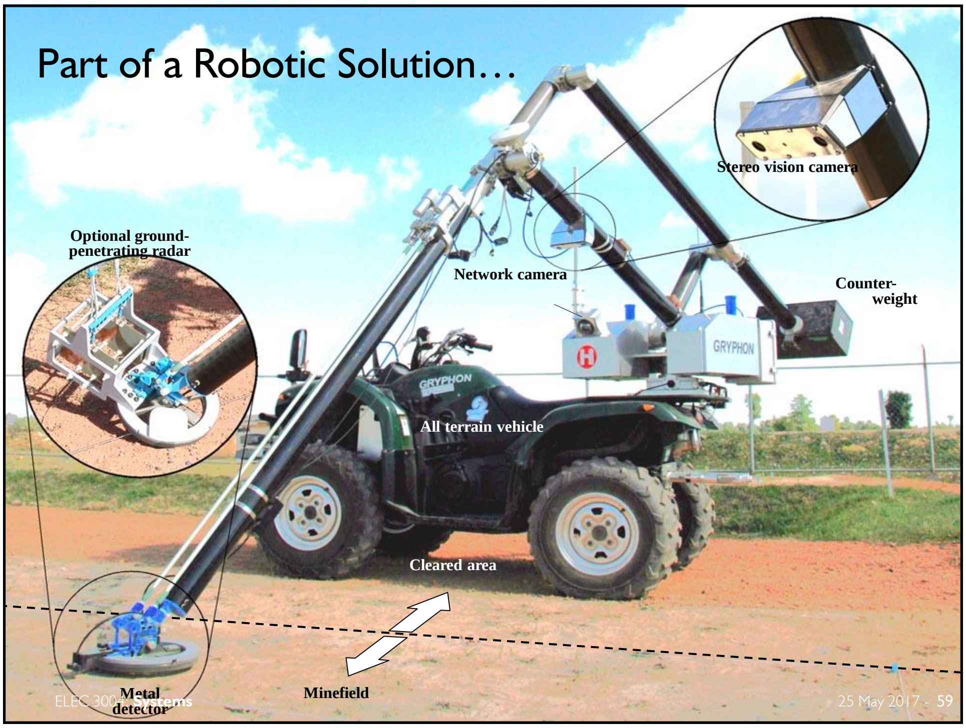
$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{(1+K)^2} & \frac{2K}{(1+K)^2} & \frac{K^2}{(1+K)^2} \\ 0 & \frac{T_d}{2} & T_d \end{bmatrix}$$

For Gryphon:

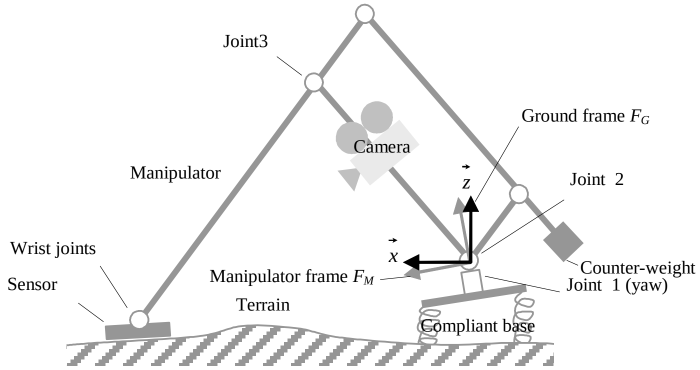
$$\begin{bmatrix} \omega \\ \zeta \end{bmatrix} = \begin{bmatrix} \omega_{\rho 0} \\ \zeta_{\rho 0} \end{bmatrix} \left(1 - \frac{\rho - \rho_0}{\rho_1 - \rho_0}\right) + \begin{bmatrix} \omega_{\rho 1} \\ \zeta_{\rho 1} \end{bmatrix} \left(\frac{\rho - \rho_0}{\rho_1 - \rho_0}\right)$$

		At ρ_0=1.5 [m]	At ρ_1=3.0 [m]
Axis 1	ω	2.32	1.81
	ζ	0	0
Axis 2 & 3	ω	3.3	3.0
	ζ	0	0

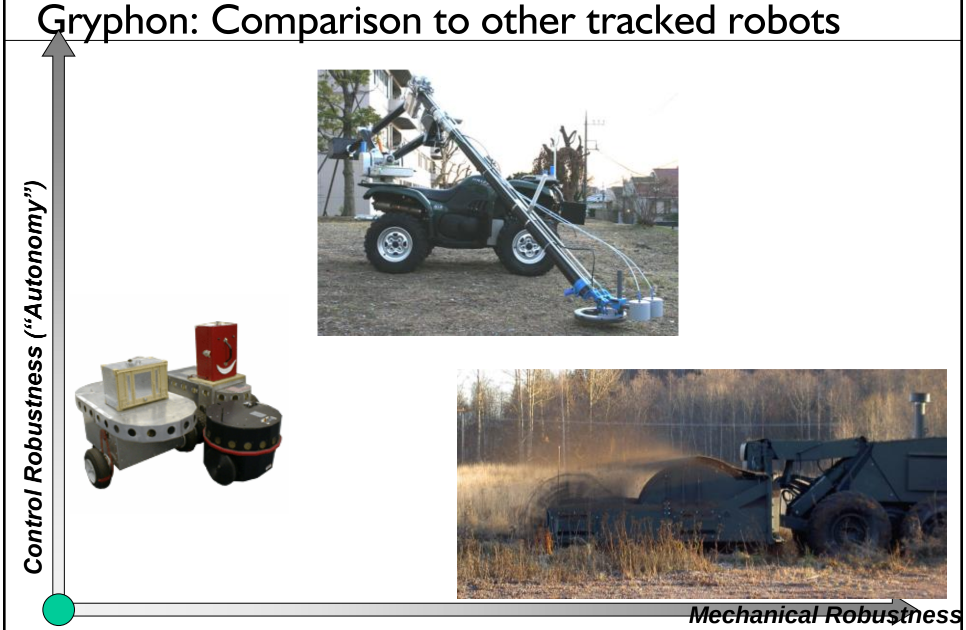
Part of a Robotic Solution…



Gryphon Schematic

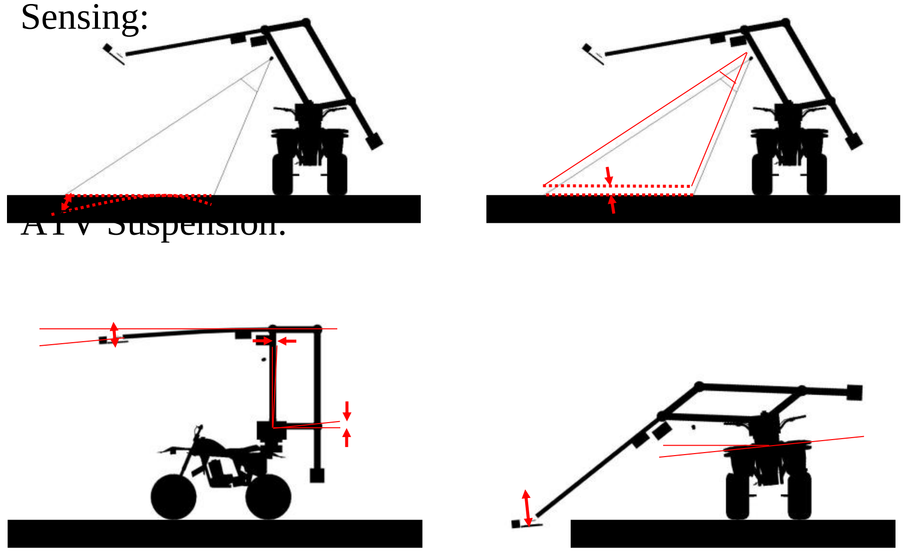


Gryphon: Comparison to other tracked robots



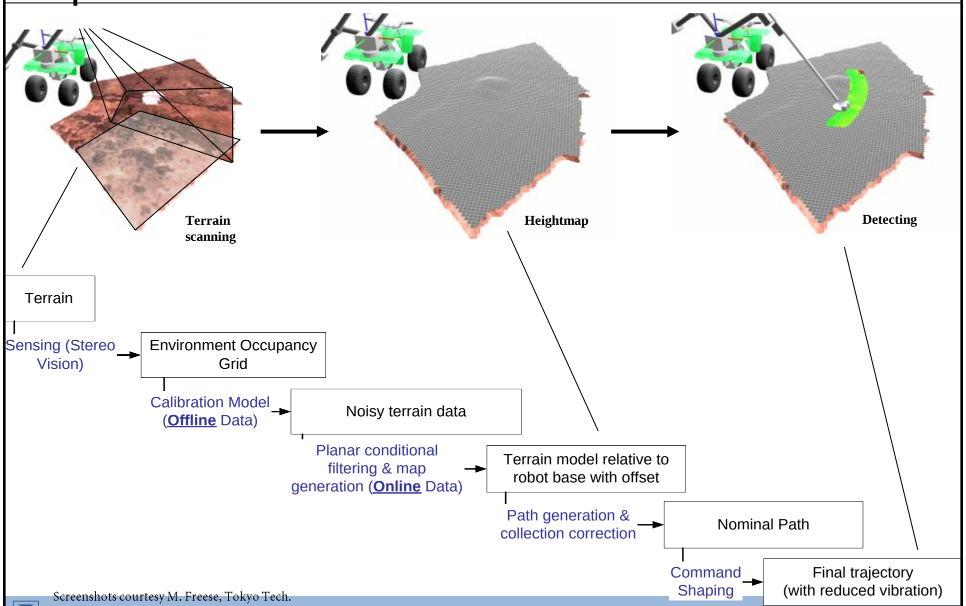
Multiple Inaccuracies

- Sensing:
- ATV Suspension:



Screenshots courtesy M. Freese, Tokyo Tech.

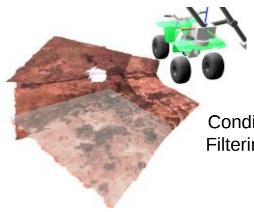
Operational Overview



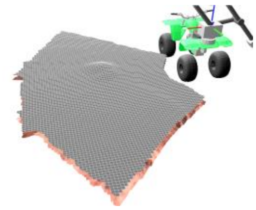
Screenshots courtesy M. Freese, Tokyo Tech.

Terrain Modeling & Following Overview

- I. Terrain Mapping
- II. Terrain Model

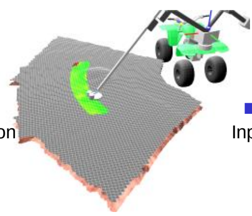


Conditional Planner
Filtering & Stitching

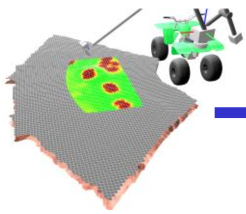


- III. Path Generation
- IV. Scanning

Height map expansion
& Path generation



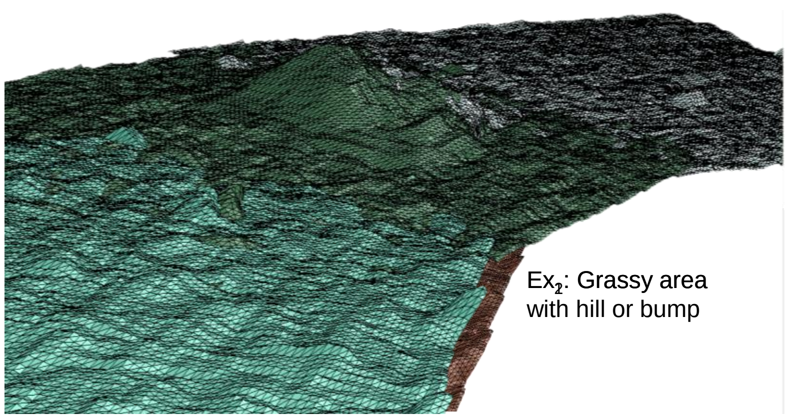
Input-shaping



V. Evaluation
& Marking

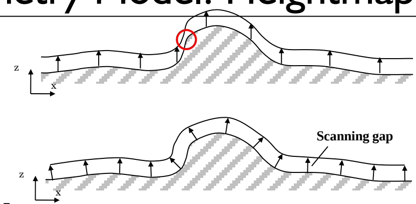
Terrain Mapping

- Stereo depth maps (Pont Gray Bumblebee)
- Kinematic calibration corrections

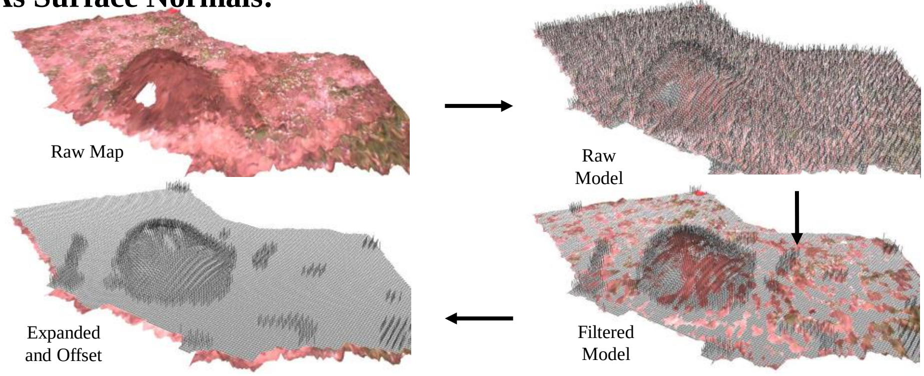


Ex$_2$: Grassy area
with hill or bump

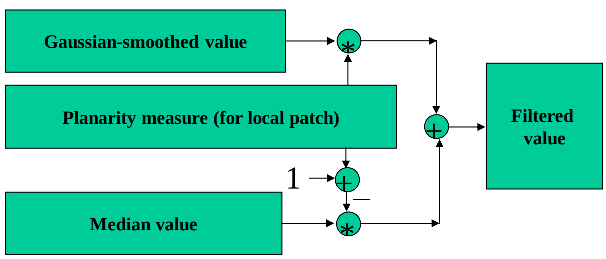
Terrain Geometry Model: Heightmap Expansion



As Surface Normals:

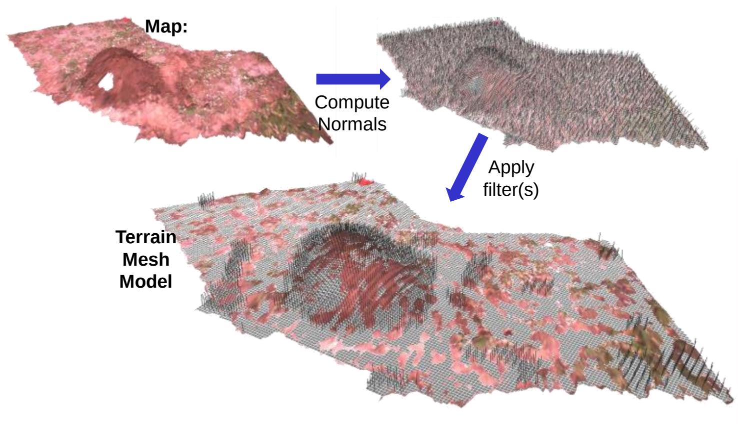


Terrain Geometry Model: Conditional Planar Filter



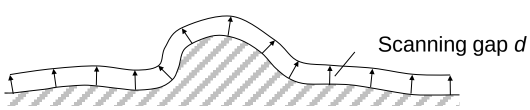
- Planarity: Found from plane eq. residuals for a surface patch
- Filter type and strength varied based on this
- Goal: Reduce noise without feature degradation

Terrain Map→Model: Conditional Planar Filter

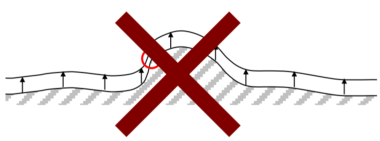


Map → Model (II): Height Map Expansion

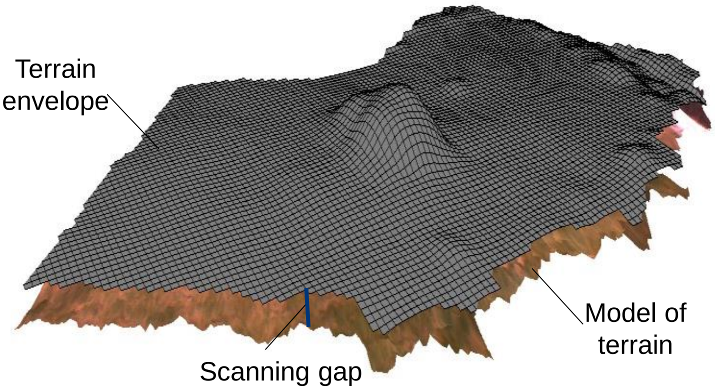
- Envelope expansion:
  - $F_{env} = F_{terr}$ + scanning gap …



  - Performed along the normals, more than vertical axis addition:

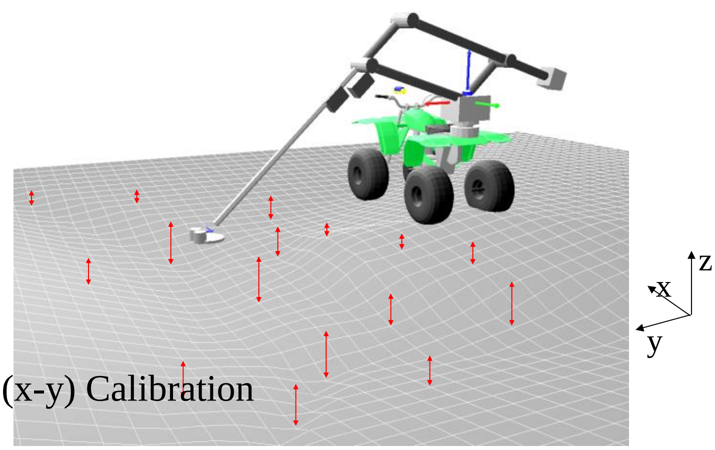


Map → Model (III): Height Map Expansion



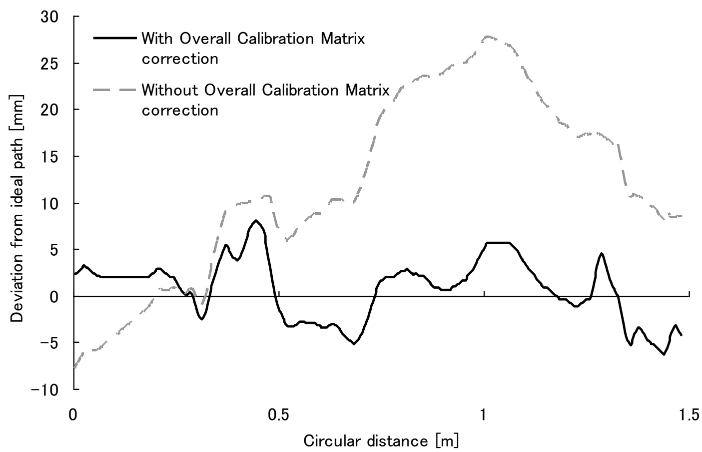
Calibration Model

- Height (z) Calibration:



- Plane (x-y) Calibration

Effect of Overall Calibration Matrix

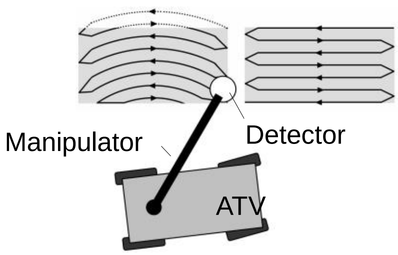


Scanning speed: 100 mm/s
Scanning gap: 100 mm

Path Generation

- x-y: Scanning Scheme
- Joint-space/Work-space?
- Reduce excess work …

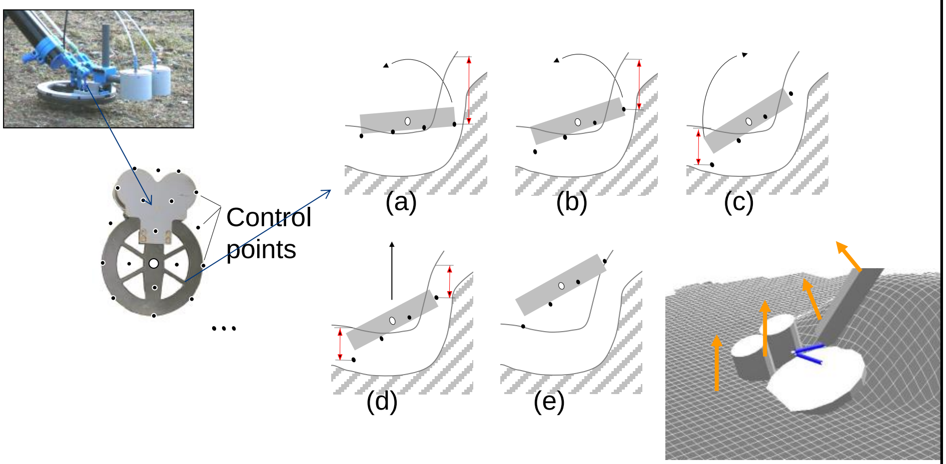
- z: Terrain Sampling (z)
- Sample corresponding point based on the local patch & normals



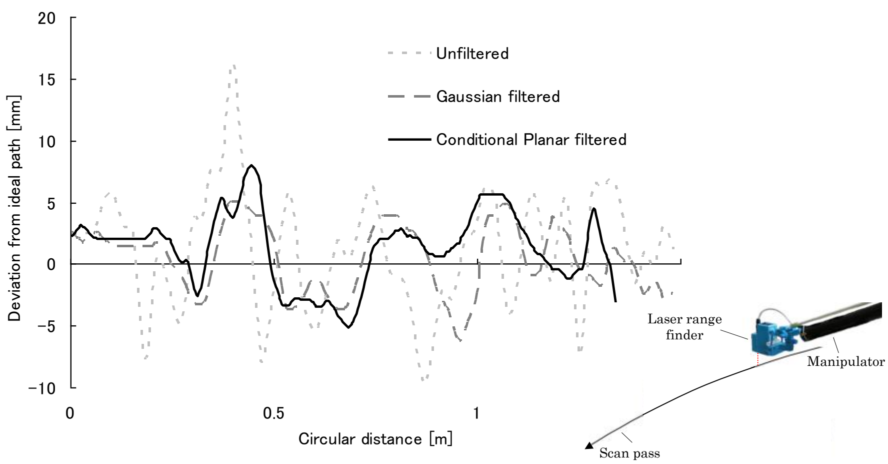
$$z_{path} = f_{env}(x_{path}, y_{path})$$

Path Generation (II)

- Orientation: Advanced Terrain Following

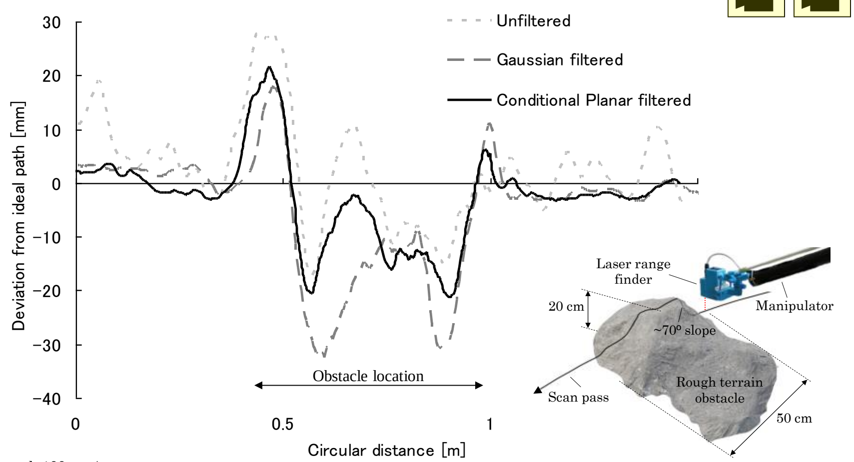


Scanning on ~ Level Terrain - Measurements



Scanning speed: 100 mm/s
Scanning gap: 100 mm

Scanning on Rough Terrain - Measurements

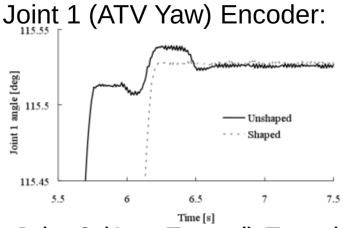


Scanning speed: 100 mm/s
Scanning gap: 100 mm

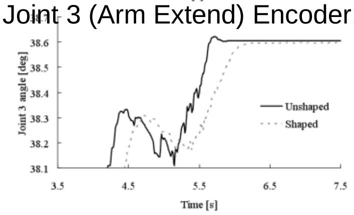
Command Shaping Tests: Step-Response

- Reduced Joint Encoder Vibration

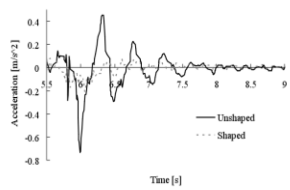
Joint 1 (ATV Yaw) Encoder:



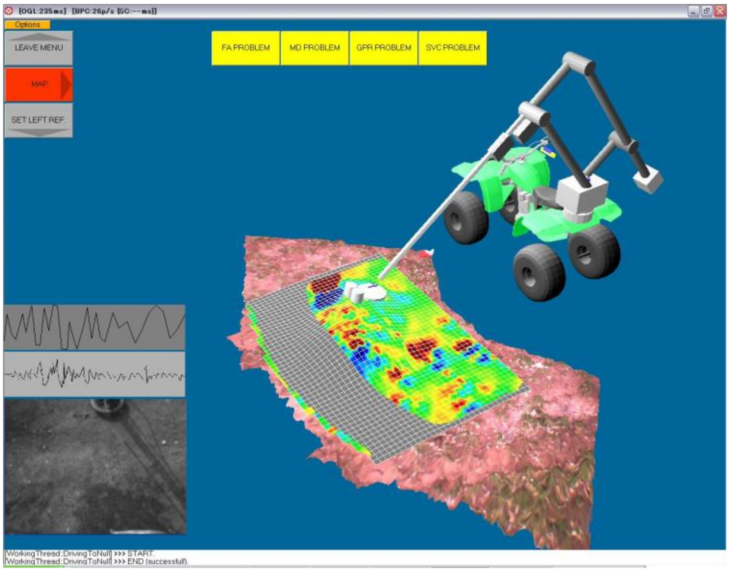
Joint 3 (Arm Extend) Encoder:



- Reduced Tip Acceleration



High-Level Control Software

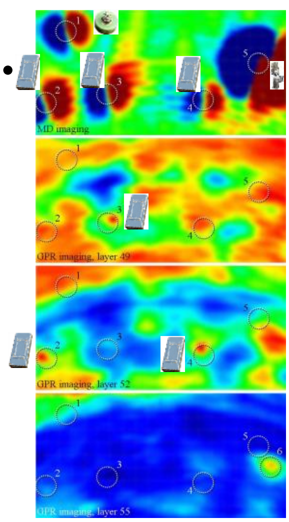


Detector Imaging

- Targets

PMA-1A PMA-2 Fragment

Target#	Target type	Depth [cm]	MD	GPR
1	PMA-2	5	Yes	No
2	PMA-1A	12.5	Yes	Yes
3	PMA-1A	12.5	Yes	Yes
4	PMA-1A	12.5	Yes	Yes
5	Fragment	5	Yes	No
6	Stone	~10	No	Yes



Gryphon: Field Tests in Croatia & Cambodia

End on a "Bang, Bang"…

A Better (Controlled) "Bang Bang"

Announcement from the UQ Central EXAMINATIONS DEPARTMENT !

(AKA: ***create change*** [in the lecture])

Are you prepared for your examinations?

Do you:

1. Have your current UQ student ID card?
2. Know where your examination is being held?
3. Know what materials you are permitted to bring to the examination? (check with your course coordinator)
4. Have an approved / labelled calculator (in exams where calculators are permitted)

Are you prepared for your examinations?

For each examination, ensure you:

1. Have rechecked your personalised examination timetable for date, time and venue
2. Have your current UQ student ID card on hand and be ready to present on entry to the examination venue – should you forget it, you must report to the Student Centre before your examination
3. Have spare pencils and pens, as well as any permitted materials
4. Arrive at your examination venue 15 minutes before the scheduled start and 30 minutes if the examination is held at the UQ Centre

Examination Timetable

Students are provided with a personalised examination timetable to their UQ email account, detailing their;

- Schedule of examinations
- Date, start time and exam duration
- Campus and specific venue to which they must attend for each examination

It is important that students;

- Attend to the venue listed on their examination timetable 15-30 minutes before the exam is due to commence
- Are in possession of their UQ Student ID card
- Have an approved calculator should it be permitted for the examination

Student ID Card – Essential!

All Students MUST present a current UQ Student ID card to gain entry to the examination venue

Approved Calculators in Examinations

With the exception of the Casio fx-82 series, all calculators used during an exam must have an official "Approved" label.

Labels are available from the Student Centre.

Calculators must be approved in advance of the examination period.

Check out my.UQ (https://my.uq.edu.au/information-and-services/manage-my-program/exams-and-assessment/sitting-exam/approved-calculators)

Mobile Phones in Examinations

Sick during Examinations Please note!!

A student ***who attends and attempts whole or part of the original examination*** *will not be eligible for a deferred examination.*

So how could this ruling affect you if you are unwell?

Either: **Commence and finish your examination;** or

Do not attend your examination, obtain a medical certificate from your doctor and apply for a Deferred Examination.

Check your eligibility criteria before you make a decision!
Please see the my.UQ website (https://my.uq.edu.au/services/exams-and-assessment) or ask at the Student Centre for assistance.

Students will be permitted to enter the examination venue until one hour into the examination (eg 08:00am exam start, entry until 09:00am).

Students will not be permitted to leave an examination until one hour of the examination has elapsed (eg 08:00am exam start, leave at 09:00am).

Students will not be permitted to wear watches in examinations.

Watches may be placed under the student's chair or on the corner of the student's desk for the duration of the examination.

For further information on -

- Taking materials into the exam room
- Arriving at an exam venue
- What if I become sick during an exam?
- Leaving the exam room
- Finishing an exam

Refer to my.UQ (https://my.uq.edu.au/information-and-services/manage-my-program/exams-and-assessment/sitting-exam)

Back to your regularly scheduled lecture

Next Time…

- **Digital Feedback Control**

- Review:
  - Chapter 2 of FPW

- More Pondering??