THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

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Family Name _____________________

First Name _____________________

School of Information Technology and Electrical Engineering

EXAMINATION

Semester One Final Examinations, 2017

ELEC3004 Signals, Systems & Control

This paper is for St Lucia Campus students.

Examination Duration: 180 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - specified materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Any calculator permitted - unrestricted

One A4 sheet of handwritten or typed notes double sided is permitted

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

1 x 1cm x 1cm Graph Paper

Instructions To Students:

Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.

Please answer all questions. Thank you! :-)

For Examiner Use Only

Q	Mark
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Total ________

⇨ **PLEASE RECORD ALL ANSWERS** ⇦
⇨ **IN THE ANSWER BOOKLET** ⇦

Any material not in Answer Booklet(s) **will not be seen**. In particular, the exam paper **will not be graded** or reviewed.

(Otherwise the rest of page is intentionally left blank – feel free to use as scratch paper)

This exam has THREE (3) Sections for a total of 180 Points
(which very roughly, on the whole, corresponds to ~1 Point/Minute)

Section 1: Digital Linear Dynamical **Systems**..60 Points (33 %)
Section 2: Digital Processing/Filtering of **Signals**................................. 60 Points (33 %)
Section 3: Digital & State-Space **Control**...60 Points (33 %)

⇨ Please answer **ALL** questions + **ALL Answers MUST Be Justified** ⇦
(answers alone are **not** sufficient)

⇨ **PLEASE RECORD ALL ANSWERS IN THE ANSWER BOOKLET** ⇦

(Any material not in Answer Booklet(s) **will not be seen**.
In particular, the exam paper **will not be graded** or reviewed.)

Section 1: Digital Linear Dynamical Systems

Please Record Answers in the **Answer Booklet** (**5 Questions** | **60 Points**)
Please **Justify and Explain All Answers**

1. **One Small Step…** (10 Points)
A basic discrete-time signal is a the discrete-time unit step, $u[n]$, such that

$$u[n] = \begin{cases} 0, n < 0 \\ 1, n \geq 0 \end{cases}$$

A. There is a close relationship between the discrete-time unit step, $u[n]$, and the discrete-time unit impulse, $\delta[n]$. Briefly, what is it?
(i.e., $\delta[n] =$ ________________________________ ?) [answer in booklet please]

B. The unit step also helps describe the units step response, $s[n]$, for a discrete-time LTI system, $h[n]$. Briefly, what is it?
(i.e., $s[n] =$ ________________________________ ?) [answer in booklet please]

C. Interestingly, for the system in B (above), the $h[n]$ can be recovered from $s[n]$ via a simple relationship. Briefly, what is it?
(i.e., $h[n] =$ ________________________________ ?) [answer in booklet please]

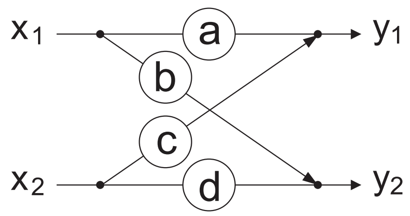
2. **A Fast and E-$\mathcal{Z}$ Fourier** (10 Points)
Given a discrete-time **unit impulse response** with the following difference equation:

$$\boldsymbol{y[n] = 3 \cdot \delta[n - 4] - \delta[n + 0] - \delta[n + 7] + 4 \cdot \delta[n + 2]}$$

A. What is its Z-transform?
(i.e., what is **Y(z)**)?

B. What is its frequency (or Fourier) response?
(i.e., what is **Y(ω)**)
[hint: for partial credit, you may leave it in terms of the phasor $e^{j\omega}$]

3. **Flowing Into Signals** (10 Points)
A signal flow graph or block diagram may be represented as a linear system ($y = Ax$). For the scalars x_1, x_2, y_1, y_2, a, b, c, d in the case illustrated below:



   A. What are the terms y, $\boldsymbol{A}$ and x as a function of the scalars

   B. If $\boldsymbol{A}$ is upper-diagonal, what can we immediately say about the dependence (or conversely independence) of y_2?

4. **Positively Inverted** (15 Points)
Consider a square matrix $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ whose inverse exists and is denoted by $\boldsymbol{B}$. Now suppose that $\boldsymbol{A}$ and $\boldsymbol{B}$ both have all their elements nonnegative (i.e., $A_{ij} \geq 0$, $B_{ij} \geq 0$ for all $i, j = 1, \ldots, n$).

   A. What is the dimension of $\boldsymbol{B}$?

   B. For the case of $n = 2$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
   What can we say of the relationship between the elements a, b, c, and d.

   C. For the general case (i.e., n is a natural number), what properties hold for **A** and **B**? (i.e., what specifically must be must be true)

5. **A Fine Little Technical Inquiry** (15 Points)
An affine linear time invariant (LTI) system may be described as one such that a function f with $f\colon \mathbb{R}^n \rightarrow \mathbb{R}^m$, $x, y \in \mathbb{R}^n$, and $\alpha, \beta \in \mathbb{R}$ with $(\alpha + \beta) = 1$ has

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$$

   A. Is an arbitrary affine system a linear system?

   B. Is an arbitrary linear system an affine system?

   C. Suppose that $\boldsymbol{A} \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$, please prove that the function $f(x) = \boldsymbol{A}x + b$ is affine

Section 2: Digital Processing & Filtering of Signals

Please Record Answers in the **Answer Booklet** (**5 Questions** | **60 Points**)
Please **Justify and Explain All Answers**

6. **A Minimum Sample** (10 Points)
Consider the signal

$$y(t) = 3cos(2\pi t) + cos\big(3\pi(t + 0.5)\big) + 4sin(5\pi t)$$

A. What is the minimum sampling frequency ω_s (assuming $\omega_s = n\pi$) and number of samples N_s that will allow resolution of all the frequencies and their perfect reconstruction?

B. If you were to reconstruct with an ideal lowpass filter, what cut-off frequency should it have?

7. **Smooth as Butter!** (10 Points)
The frequency response magnitude of a normalised Butterworth filter of order n is given by:

$$|H_n(\omega)| = \frac{1}{\sqrt{1 + \omega^{2n}}}$$

A. Please determine the transfer function of a 2nd-order, **high-pass** Butterworth filter with cut-in frequency equal to 6 kHz.

B. At what frequency is the gain of this filter $-3\ dB$? $-30dB$?

8. **Reading Between the Lines** (10 Points)
For the sequence

$$x[n] = \{0,3,0,0,4,0,-7,2,0\}$$

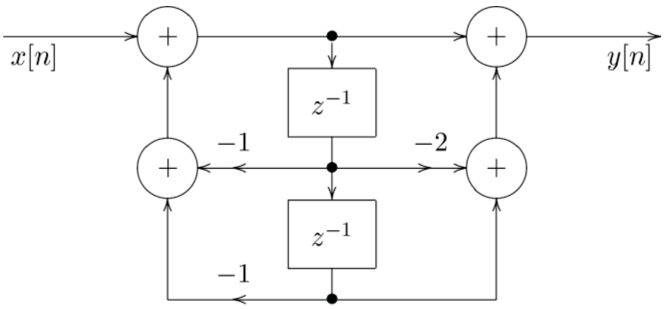
And, an **up-sampling** factor of $L = 2$, please reconstruct the output sequence $x(t)$ via:

A. A Zero Order Hold (ZOH) interpolation approach

B. A Whittaker–Shannon ($sinc$) interpolation approach

[**Hint**: a function **or** a plot is acceptable.
If a plot is provided, **then** please label the axes]

9. **Squarely Digital Filters** (15 Points)
Consider the digital filter given by:



For this filter, please calculate the following:

A. Is it IIR or FIR?

B. The difference equation

C. The first four (4) samples of the impulse response

D. The gain at DC (i.e., $\omega = 0$)

E. The gain at the Nyquist frequency (i.e., $\omega = \frac{\omega_s}{2}$)

10. **Sound Sampling** (15 Points)
Inspired by an Irish ballad, Mr Crooner plans to quit his job and form the band Me2. He plans to record his songs digitally to compact disc (CD). Assume that the songs have frequencies ranging from 100 Hz-1,100 Hz (i.e. 1.1 kHz).

A. In this case, what is the Nyquist rate?

B. Briefly, what is an assumption about the clock/sampler for digital signals?

C. A CD actually uses 44,100 samples/s. If the samples are quantized into 65,536 levels ($L = 65{,}536$), determine the number of binary digits required to encode a 3:14 (3 minute and 14 second) long song?

D. Mr. Crooner's agent, Stephen Cacophonous, has no time for so many samples and wants to record songs using 2,000 samples/sec instead.
What will happen? Will there be aliasing? If so, what frequencies will alias?

E. Besides adding an additional anti-aliasing filter, suggest a way that aliasing (if it exists) can be removed from the above case (in D) without changing the sample rate (from 2,000 samples/sec).

Section 3: Digital & State-Space Control

Please Record Answers in the **Answer Booklet** (**5 Questions** | **60 Points**)
Please **Justify and Explain All Answers**

11. **An Impulsive Response** (10 Points)

Stephen Cacophonous, the ever impulsive and hapless manager, is now going try his hand at directing – this time with Shakespearean flair. Taking inspiration from *Much Ado About Nothing's* "I wish my horse had the speed of your tongue" [I:1, Benedick] he seeks to rule the show (and this exam) with haste and needs help getting impulse responses. Systems, like Stephen (and Beatrice's tongue), can be rather higgledy-piggledy when tested impulsively.

A. **Continuous Time**. Consider the input signal given by

$$x(t) = 4\delta(t-1)$$

For a system whose impulse response is given by

$$h(t) = 2[\delta(t) + \delta(t-4)]$$

For this please determine the output $y(t)$
[**Hint**: a function **or** a plot is acceptable. **If** a plot is provided, **then** please label the axes]

B. **Discrete Time.** Consider the input sequence

$$x[n] = \{1,0,1,1\}$$

That is input to a LTI system whose impulse response is given by:

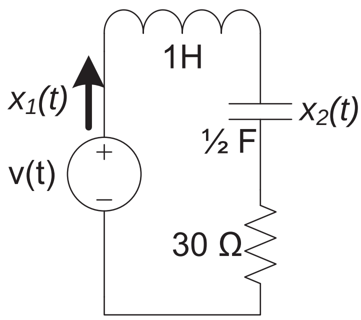
$$h[n] = \{3,2,1\}$$

For this, please calculate the output, $y[n]$.

[**Hint**: a function **or** a plot is acceptable. **If** a plot is provided, **then** please label the axes]

12. **A Stately Circuit** (10 Points)

Consider the simple RLC circuit given below with inductor current, $x_1(t)$, capacitor voltage, $x_2(t)$, and input supply $v(t)$.

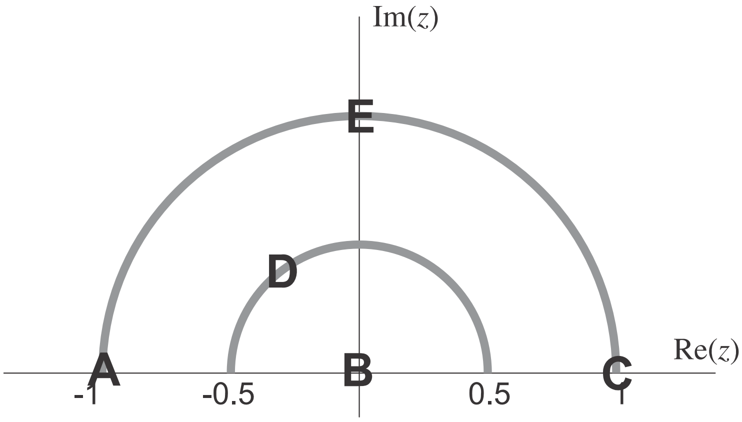


A. Please express every voltage and current as a linear combination of $x_1(t)$, $x_2(t)$ and $v(t)$.

B. Please express this in state-space.
(i.e., as a linear system for the internal state vector $\boldsymbol{x}(t)$)

13. **Images of the $\mathcal{Z}$-Plane** (10 Points)

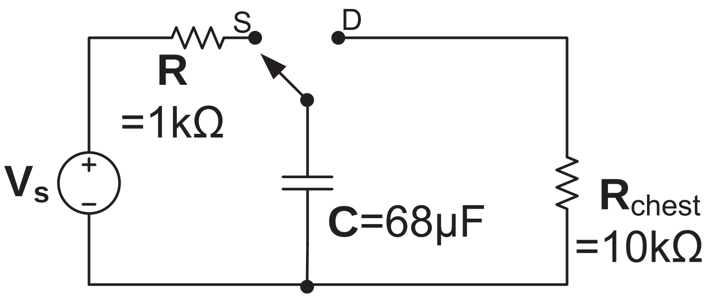
For a first order system with a pole at the locations A, B, C, D, or E as indicated the following diagram of the $\mathcal{Z}$-Plane



Please briefly sketch the time response associated with each of these locations (i.e., you should have 5 small sketches for the typical signal response at each location marked **A, B, C, D,** and **E**.)

14. **I ♥ Systems** (15 Points)

A defibrillator is used to deliver a strong shock across the chest of a person in cardiac arrest (i.e., fibrillation). A simple design for one may be constructed using the following circuit:



With the switch in the standby mode ("S"), the 68μF capacitor is charged using a controller (having a Thevenin equivalent of $\mathbf{V_s}$ and **R**=1kΩ). To defibrillate, the switch is thrown (to "D") and the capacitor discharges across the patient's chest, which can be approximated as a 10kΩ resistor.

A. Determine $\mathbf{V_s}$ so that the dose is 150J
(Assume the capacitor is fully charged when the switch is thrown)?

B. Using this value, specify how long it takes (in seconds) to deliver 95J?

15. **A Colourful Ending**[1] (15 Points)

Human colour perception is performed colour reception cells, called *cones*. Each type of cone has a different spectral response characteristic. That is, a R, G and B response for "red" (long λ), "green" (medium λ), and "blue" (short λ), respectively.

For simplicity, one can divide the visible spectrum into 10 bands, and model the cones' response as follows:

$$R_{cone} = \sum_{i=1}^{10} r_i p_i \qquad G_{cone} = \sum_{i=1}^{10} g_i p_i \qquad B_{cone} = \sum_{i=1}^{10} b_i p_i$$

where p_i is the incident power in the i^{th} wavelength band, and r, g_i and b_i are non-negative constants that describe the spectral response of the different cones. The sensed colour is complex/vector function of the three cone responses.

A. Determine (non-trivially) when two light spectra, p and $\hat{p}$ are visually indistinguishable? (This may be a specific case or, preferably, a general rule)
[**Note**: Visually identical cases with different spectral power compositions are called *metamers*]

B. In a colour matching operation, an observer is shown a test light and is asked to change the intensities of three primary lights until the sum of the primary lights looks the same. That is, to find a spectrum of the form $p_{match} = a_1\lambda_u + a_2\lambda_v + a_3\lambda_w$, where λ_u, λ_v, λ_w are the spectra of the primary lights, and a_i are the intensities that, if/when found, are visually indistinguishable from a test light spectrum p_{test}. Can this always be done? **Please briefly discuss.**

C. **Final Illumination**. An object's surface can be characterized by its reflectance (i.e., the fraction of light it reflects) for each band of wavelengths (λ).

Now consider two objects illuminated (at different times) by two different light sources (e.g., LED lights and sunlight). Stephen (Cacophonous) argues that if the two objects look identical when illuminated by LED lights, they will look identical when illuminated by sunlight. Hannah disagrees and says that two objects can appear identical when illuminated by LED lights, but look different when lit by sunlight.

Who is right? If Stephen is right, then explain why. If Hannah is right, then give two (2) examples of objects that appear identical under one light source and different under another.

END OF EXAMINATION — Thank you !!!

From Brisbane to Perth,
May the Wonder of Linear Dynamical Systems Take You Places & Bring Mirth ☺

[1] For more information see "Color Perception" in S. Boyd, *Lecture Notes for EE263*, 2012. (After the exam, of course)

Table 1: Commonly used Formulae

The Laplace Transform

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt$$

The $\mathcal{Z}$ Transform

$$F(z) = \sum_{n=0}^{\infty} f[n]z^{-n}$$

IIR Filter Pre-warp

$$\omega_a = \frac{2}{\Delta t}\tan\left(\frac{\omega_d \Delta t}{2}\right)$$

Bi-linear Transform

$$s = \frac{2(1-z^{-1})}{\Delta t(1+z^{-1})}$$

FIR Filter Coefficients

$$c_n = \frac{\Delta t}{\pi}\int_0^{\pi/\Delta t} H_d(\omega)\cos(n\omega\Delta t)\,d\omega$$

Table 2: Comparison of Fourier representations.

Time Domain	Periodic	Non-periodic	
Discrete	**Discrete Fourier Transform** $\tilde{X}[k] = \frac{1}{N}\sum_{n=0}^{N-1}\tilde{x}[n]e^{-j2\pi kn/N}$ $\tilde{x}[n] = \sum_{k=0}^{N-1}\tilde{X}[k]e^{j2\pi kn/N}$	**Discrete-Time Fourier Transform** $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)e^{j\omega n}\,d\omega$	**Periodic**
Continuous	**Complex Fourier Series** $X[k] = \frac{1}{T}\int_{-T/2}^{T/2}\tilde{x}(t)e^{-j2\pi kt/T}\,dt$ $\tilde{x}(t) = \sum_{k=-\infty}^{\infty} X[k]e^{j2\pi kt/T}$	**Fourier Transform** $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$ $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$	**Non-periodic**
	Discrete	**Continuous**	**Freq. Domain**

Table 3: Selected Fourier, Laplace and z-transform pairs.

Signal	$\longleftrightarrow$	Transform	ROC
$\tilde{x}[n] = \sum_{p=-\infty}^{\infty} \delta[n - pN]$	$\overset{DFT}{\longleftrightarrow}$	$\tilde{X}[k] = \frac{1}{N}$	
$x[n] = \delta[n]$	$\overset{DTFT}{\longleftrightarrow}$	$X(e^{j\omega}) = 1$	
$\tilde{x}(t) = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$\overset{FS}{\longleftrightarrow}$	$X[k] = \frac{1}{T}$	
$\delta_T[t] = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$	
$\cos(\omega_0 t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$	
$\sin(\omega_0 t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = j\pi\delta(\omega + \omega_0) - j\pi\delta(\omega - \omega_0)$	
$x(t) = \begin{cases} 1 & \text{when } \lvert t\rvert \leqslant T_0, \\ 0 & \text{otherwise.} \end{cases}$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \frac{2\sin(\omega T_0)}{\omega}$	
$x(t) = \frac{1}{\pi t}\sin(\omega_c t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \begin{cases} 1 & \text{when } \lvert\omega\rvert \leqslant \lvert\omega_c\rvert, \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = 1$	
$x(t) = \delta(t - t_0)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = e^{-j\omega t_0}$	
$x(t) = u(t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \pi\delta(w) + \frac{1}{jw}$	
$x[n] = \frac{\omega_c}{\pi} \operatorname{sinc} \omega_c n$	$\overset{DTFT}{\longleftrightarrow}$	$X(e^{j\omega}) = \begin{cases} 1 & \text{when } \lvert\omega\rvert < \lvert\omega_c\rvert, \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = 1$	all s
(unit step) $x(t) = u(t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{1}{s}$	
(unit ramp) $x(t) = t$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{1}{s^2}$	
$x(t) = \sin(s_0 t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{s_0}{(s^2 + {s_0}^2)}$	
$x(t) = \cos(s_0 t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{s}{(s^2 + {s_0}^2)}$	
$x(t) = e^{s_0 t}u(t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{1}{s - s_0}$	$\mathfrak{Re}\{s\} > \mathfrak{Re}\{s_0\}$
$x[n] = \delta[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = 1$	all z
$x[n] = \delta[n - m]$	$\overset{z}{\longleftrightarrow}$	$X(z) = z^{-m}$	
$x[n] = u[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{z}{z - 1}$	
$x[n] = z_0^n u[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{1}{1 - z_0 z^{-1}}$	$\lvert z\rvert > \lvert z_0\rvert$
$x[n] = -z_0^n u[-n - 1]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{1}{1 - z_0 z^{-1}}$	$\lvert z\rvert < \lvert z_0\rvert$
$x[n] = a^n u[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{z}{z - a}$	$\lvert z\rvert < \lvert a\rvert$

Table 4: Properties of the Discrete-time Fourier Transform.

Property	Time domain	Frequency domain
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\omega}) + bX_2(e^{j\omega})$
Differentiation (frequency)	$nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
Time-shift	$x[n-n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
Frequency-shift	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
Convolution	$x_1[n] * x_2[n]$	$X_1(e^{j\omega})X_2(e^{j\omega})$
Modulation	$x_1[n]x_2[n]$	$\frac{1}{2\pi}X_1(e^{j\omega}) \circledast X_2(e^{j\omega})$
Time-reversal	$x[-n]$	$X(e^{-j\omega})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(e^{j\omega}) = X^*(e^{-j\omega})$
Symmetry (imag)	$\mathfrak{Re}\{x[n]\} = 0$	$X(e^{j\omega}) = -X^*(e^{-j\omega})$
Parseval	$\displaystyle\sum_{n=-\infty}^{\infty} \|x[n]\|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left\|X(e^{j\omega})\right\|^2 d\omega$	

Table 5: Properties of the Fourier series.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1[k] + bX_2[k]$
Differentiation (time)	$\dfrac{d\tilde{x}(t)}{dt}$	$\dfrac{j2\pi k}{T}X[k]$
Time-shift	$\tilde{x}(t-t_0)$	$e^{-j2\pi k t_0/T}X[k]$
Frequency-shift	$e^{j2\pi k_0 t/T}\tilde{x}(t)$	$X[k-k_0]$
Convolution	$\tilde{x}_1(t) \circledast \tilde{x}_2(t)$	$TX_1[k]X_2[k]$
Modulation	$\tilde{x}_1(t)\tilde{x}_2(t)$	$X_1[k] * X_2[k]$
Time-reversal	$\tilde{x}(-t)$	$X[-k]$
Conjugation	$\tilde{x}^*(t)$	$X^*[-k]$
Symmetry (real)	$\mathfrak{Im}\{\tilde{x}(t)\} = 0$	$X[k] = X^*[-k]$
Symmetry (imag)	$\mathfrak{Re}\{\tilde{x}(t)\} = 0$	$X[k] = -X^*[-k]$
Parseval	$\displaystyle\frac{1}{T}\int_{-T/2}^{T/2} \|\tilde{x}(t)\|^2 dt = \sum_{k=-\infty}^{\infty} \|X[k]\|^2$	

Table 6: Properties of the Fourier transform.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$
Duality	$X(jt)$	$2\pi x(-\omega)$
Differentiation	$\dfrac{dx(t)}{dt}$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{j\omega}X(j\omega) + \pi X(j0)\delta(\omega)$
Time-shift	$x(t-t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency-shift	$e^{j\omega_0 t}x(t)$	$X(j(\omega-\omega_0))$
Convolution	$x_1(t) * x_2(t)$	$X_1(j\omega)X_2(j\omega)$
Modulation	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(j\omega) * X_2(j\omega)$
Time-reversal	$x(-t)$	$X(-j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Symmetry (real)	$\mathfrak{Im}\{x(t)\} = 0$	$X(j\omega) = X^*(-j\omega)$
Symmetry (imag)	$\mathfrak{Re}\{x(t)\} = 0$	$X(j\omega) = -X^*(-j\omega)$
Scaling	$x(at)$	$\dfrac{1}{\lvert a\rvert}X\left(\dfrac{j\omega}{a}\right)$
Parseval	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt = \dfrac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\,d\omega$	

Table 7: Properties of the z-transform.

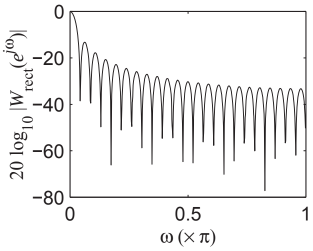
Property	Time domain	z-domain	ROC
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Time-shift	$x[n-n_0]$	$z^{-n_0}X(z)$	$R_x^\dagger$
Scaling in z	$z_0^n x[n]$	$X(z/z_0)$	$\lvert z_0\rvert R_x$
Differentiation in z	$nx[n]$	$-z\dfrac{dX(z)}{dz}$	$R_x^\dagger$
Time-reversal	$x[-n]$	$X(1/z)$	$1/R_x$
Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(z) = X^*(z^*)$	
Symmetry (imag)	$\mathfrak{Re}\{X[n]\} = 0$	$X(z) = -X^*(z^*)$	
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Initial value	$x[n] = 0, n < 0 \Rightarrow x[0] = \lim_{z\to\infty} X(z)$		

† $z = 0$ or $z = \infty$ may have been added or removed from the ROC.

Table 8: Commonly used window functions.

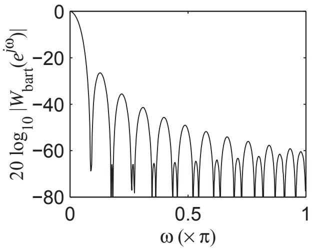
Rectangular:

$$w_{\text{rect}}[n] = \begin{cases} 1 & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



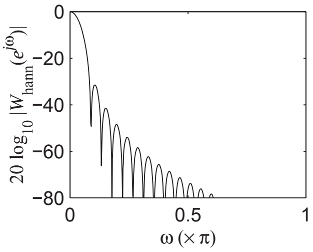
Bartlett (triangular):

$$w_{\text{bart}}[n] = \begin{cases} 2n/M & \text{when } 0 \leqslant n \leqslant M/2, \\ 2 - 2n/M & \text{when } M/2 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



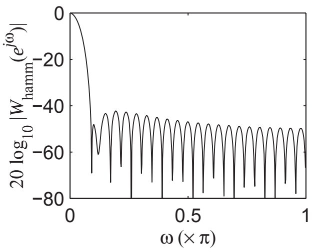
Hanning:

$$w_{\text{hann}}[n] = \begin{cases} \frac{1}{2} - \frac{1}{2}\cos(2\pi n/M) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



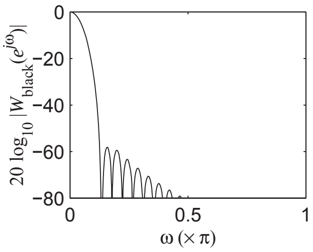
Hamming:

$$w_{\text{hamm}}[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



Blackman:

$$w_{\text{black}}[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



Type of Window	Peak Side-Lobe Amplitude (Relative; dB)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)
Rectangular	-13	$4\pi/(M+1)$	-21
Bartlett	-25	$8\pi/M$	-25
Hanning	-31	$8\pi/M$	-44
Hamming	-41	$8\pi/M$	-53
Blackman	-57	$12\pi/M$	-74

