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Seat Number	________
Student Number	|__|__|__|__|__|__|__|__|
Family Name	_____________________
First Name	_____________________

THE UNIVERSITY OF QUEENSLAND AUSTRALIA

This exam paper must not be removed from the venue

School of Information Technology and Electrical Engineering

EXAMINATION

Semester One Final Examinations, 2016

ELEC3004 Signals, Systems & Control

This paper is for St Lucia Campus students.

Examination Duration: 180 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - specified materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Any unmarked paper dictionary is permitted

An unmarked Bilingual dictionary is permitted

Calculators - Any calculator permitted - unrestricted

One A4 sheet of handwritten notes double sided is permitted

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

1 x 1cm x 1cm Graph Paper

Rough Paper

Instructions To Students:

Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.

For Examiner Use Only

Question	Mark
1	
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Total ________

⇨ **PLEASE RECORD ALL ANSWERS** ⇦
⇨ **IN THE ANSWER BOOKLET** ⇦

Any material not in Answer Booklet(s) **will not be seen**. In particular, the exam paper **will not be graded** or reviewed.

(Otherwise the rest of page is intentionally left blank – feel free to use as scratch paper)

This exam has THREE (3) Sections for a total of 180 Points
(which very roughly, on the whole, corresponds to ~1 Point/Minute)

⇨ Please answer **ALL** questions + **ALL Answers MUST Be Justified** ⇦
(answers alone are not sufficient)

⇨ **PLEASE RECORD ALL ANSWERS IN THE ANSWER BOOKLET** ⇦
(Any material not in Answer Booklet(s) **will not be seen**. In particular, the exam paper **will not be graded** or reviewed.)

Section 1: Digital Linear Dynamical Systems

Please Record Answers in the **Answer Booklet** (**5 Questions** | **60 Points**)
Please **Justify and Explain All Answers**

1. **A Sample of Truth** (10 Points)
A sampler with a sampling rate of f_s Hz may be described as a continuous time system with by the function:

$$y(t) = \sum_{k=-\infty}^{\infty} x(t)\delta(t - kT)$$

Please indicate if the following statements are either:

- Surely **TRUE**
- Surely **FALSE**
- **INDETERMINATE** (i.e. cannot be determined)

A. Sampling is Causal

B. Sampling is Invertible

C. A **Low-pass filter** with a cut-off at f_s is required for **Antialiasing**

D. Let $x(t)$ be a band-limited signal with $X(j\omega) = 0$ for $|\omega| > \omega_N$.
Then $\boldsymbol{x(t)}$ is uniquely determined by its samples $\boldsymbol{y(t)}$ if $f_S > 4\pi\omega_N$.

E. Let $x(t)$ be a band-limited signal with $X(j\omega) = 0$ for $|\omega| > \omega_N$.
Then the samples $\boldsymbol{y(t)}$ are uniquely determined if $f_S > 4\pi\omega_N$.

2. **Matrix Infer-mation** (10 Points)
Assume $\mathbf{A} \in \mathbb{R}^{m\times 4}$ and $\mathbf{B} \in \mathbb{R}^{k\times p}$ and that the operation $(\mathbf{A}^{\mathrm{T}}\mathbf{B})^{-1}$ is valid. Based on the rules of linear algebra, what can we say **exactly** about the dimensions of **A** and **B**; that is, please provide an equation or formula for m, k and p.
Please explain and be specific.

3. **A Frequently E-$\mathcal{Z}$ Problem** (10 Points)
Given a discrete-time **unit impulse response** with the following difference equation:

$$\boldsymbol{y[n] = \delta[n-3] - \delta[n+0] - \delta[n+0] + \delta[n+4]}$$

A. What is its Z-transform?
(i.e., what is **Y(z)**?)

B. What is its frequency (or Fourier) response? (i.e., what is **Y(ω)**?)
[**Hint:** for partial credit, you may leave it in terms of the phasor $e^{j\omega}$]

4. **Convoluted Convolutions?** (15 Points)
Given:

$$x_0[k] = [0 \quad 1 \quad 1 \quad 2 \quad 3 \quad 5 \quad 8]$$
$$h_1[k] = [1 \quad 0 \quad 0 \quad -1]$$
$$h_2[k] = [3 \quad 0 \quad 0 \quad 4]$$
$$y_A[k] = x_0[k] * h_1[k]$$
$$y_B[k] = x_0[k] * h_2[k]$$

Discuss whether the following statements are **surely TRUE** or **surely FALSE** or **INDETERMINATE** (cannot be determined given the information).

A. The convolution $y_A[k] = x_0[k] * h_1[k]$ is
$y_A[k] = [0 \quad 1 \quad 1 \quad 2 \quad 2 \quad 4 \quad 6 \quad -3 \quad -5 \quad -8]$

B. The convolution $y_B[k] = x_0[k] * h_2[k]$ is
$y_B[k] = [3 \quad 6 \quad 13 \quad 19 \quad 32 \quad 12 \quad 20]$

C. The distributive property holds,
so, $x_0[k] * \left[h_1[k] + h_2[k]\right] = x_0[k] * h_1[k] + x_0[k] * h_2[k]$
thus, $x_0[k] * \left[h_1[k] + h_2[k]\right] = [0 \quad 4 \quad 4 \quad 8 \quad 15 \quad 23 \quad 38 \quad 9 \quad 15 \quad 24]$

5. **Shattering Past Nyquist?** (15 Points)
A digital amplifier that can operate at a sampling rate between 2**00 and 2000 Hz** is to be used to transmit the voice of an opera singer who can precisely, but meekly, deliver A♯ (A-Sharp) notes across select "lined" octaves – chiefly at:

One-lined (A♯$_4$): 466 Hz	Two-lined (A♯$_5$): 932 Hz	Three-lined (A♯$_6$): 1865 Hz

On the other side of the transmission is a crystal glass, with a resonant frequency of **1068 Hz**.

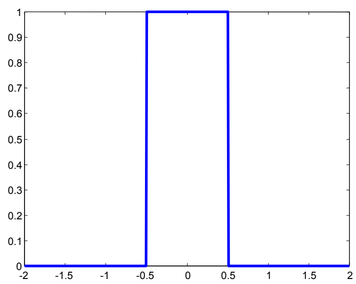
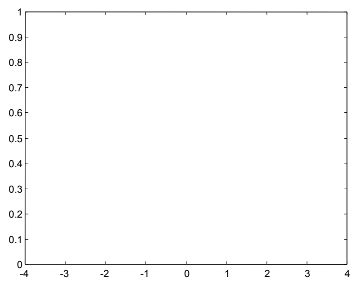
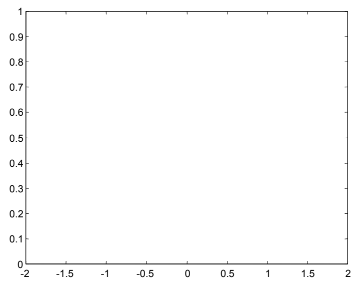
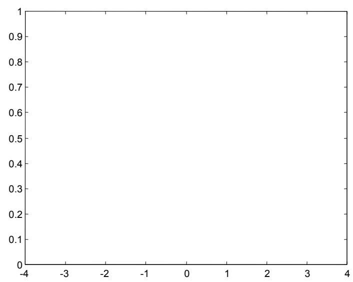
Is it possible for the glass to shatter (assume arbitrarily high amplifier power)?
If so, what **sampling rate** (f_s: [200, 2000] Hz) and **octave** (A♯$_4$, A♯$_5$ or A♯$_6$) could be used? (a specific combination **or** a general equation is acceptable).
If not, please **briefly justify** the reason.

Section 2: Digital Processing & Filtering of Signals

Please Record Answers in the **Answer Booklet** (**5 Questions** | **60 Points**)
Please **Justify and Explain All Answers**

6. **Know Noise?** (10 Points)
Additive signal noise may be characterised in many forms in the frequency spectrum.
   A. **Briefly explain** (and/or show on a simple sketch) what is meant by the terms ***white*** and ***pink (flicker)*** noise.
   B. Sometimes, multiple sensors are sometimes used to overcome sensor noise. **Briefly explain TWO (2)** assumptions needed such that averaging the values from the multiple sensors is a valid mechanism to overcome this noise.
   C. **Briefly describe** one other strategy (other than "averaging" or "integration") for beating the noise in cases with multiple sensors.

7. **Towards a Sharper Reconstruction** (10 Points)
The **Zero-Order Hold** (nearest neighbour) and **First-Order Hold** (linear) are two interpolation schemes. Please complete the following table about their impulse and frequency responses.
[**Note:** Please remember to answer in the **answer booklet**]

Response / Method	Impulse Response	Frequency Response
ZOH	 $f_{ZOH}(t) = \boxed{rect(t)}$	 $F_{ZOH}(f) = \boxed{\ ?\ }$
FOH	 $g_{FOH}(t) = \boxed{\ ?\ }$	 $G_{FOH}(f) = \boxed{\ ?\ }$

8. **IIR Filters: If I Remember Filters?** (10 Points)
Please **briefly state and compare** the main characteristics of the following recursive prototype filters:

- Butterworth
- Chebyshev (Type I and II)
- Elliptic (Cauer)

9. **An Impulsive Question** (15 Points)
As noted on Table 3 (p. 11), the $\mathcal{Z}$-transform of the exponential impulse response $x[n] = a^n u[n]$ is $X(z) = \frac{z}{z-a}$.

A. What is the magnitude and phase of $X(z)$ at 0 Hz ?

B. What is the magnitude and phase of $X(z)$ at the Nyquist rate ($\Omega=\pi$)?
[Note: where Ω is the radian frequency normalized so that the sampling rate=2π (or "1 Hz")]

C. Please sketch the magnitude and phase response of $X(z)$ over $-2\pi<\Omega<2\pi$

D. What is the inverse transform of the squared system?
That is, for $X_2(z) = [X(z)]^2$, please determine $x_2[n]$.

10. **AFIRmative Action** (15 Points)
Using the window method (with a rectangular window) find the coefficients of a causal, FIR, linear phase, digital filter of length 7 which approximates the ideal frequency response,

$$H_d(\omega) = \begin{cases} 1, & |\omega| \le \frac{\pi}{6} \\ 0, & \frac{\pi}{6} < |\omega| \le \pi \end{cases}$$

Name one advantage and one disadvantage of using a Hanning or Hamming window as compared to a rectangular window. Of these three windows (Rectangular, Hanning and Hamming), which one is **preferred for filtering** (as distinct from spectral analysis) and **briefly why**?

Section 3: Digital & State-Space Control

Please Record Answers in the **Answer Booklet** (**5 Questions | 60 Points**)
Please **Justify and Explain All Answers**

11. **Discreetly Equivalent** (10 Points)

Given the following plants (Plants A, B and C respectively):

$$G_a(s) = \frac{10s + 10}{\frac{s^2}{10} + s + \mathbf{10}}$$

$$G_b(s) = \frac{10s + 10}{\frac{s^2}{10} + s + \mathbf{100}}$$

$$G_c(s) = \frac{10s + 10}{\frac{s^2}{10} + s + \mathbf{1000}}$$

Recall the Trapezoid Rule is given by $s \to \frac{2}{T}\frac{z-1}{z+1}$

A. True or False?
The $\mathcal{Z}$ domain equivalent $G_a(z)$ found via the Trapezoid Rule is given by:

$$G_a(z) = \frac{\frac{20(z-1)}{T(z+1)} + 10}{\frac{2(z-1)}{T(z+1)} + \frac{2(z-1)^2}{5T^2(z+1)^2} + 10}$$

$$= \frac{25T(z+1)(T + 2z + Tz - 2)}{25T^2z^2 + 50T^2z + 25T^2 + 5Tz^2 - 5T + z^2 - 2z + 1}$$

B. What are the $\mathcal{Z}$ domain equivalents $G_b(z)$ and $G_c(z)$ as determined using the Trapezoid Rule?
[**Hint**: Consider the structure of Plants A, B and C, particularly their characteristic equations]

12. **Should PID be Positively In Demand?** (10 Points)

PID Control may be considered in the Z-Domain as:

$$D(z) = K_p\left(1 + \frac{Tz}{T_I(z-1)} + \frac{T_D(z-1)}{Tz}\right)$$

And/or as a difference equation as a

$$u(t_k) = u(t_{k-1}) + K_p\left[\left(1 + \frac{\Delta t}{T_I} + \frac{T_D}{\Delta t}\right)e(t_k) + \left(-1 - \frac{2T_D}{\Delta t}\right)e(t_{k-1}) + \frac{T_D}{\Delta t}e(t_{k-2})\right]$$

A. How many degrees of freedom does a PID control law have?

B. **Briefly** and **succinctly** describe what might happen to a system where $\mathbf{T_D}$ is **increased**

C. **Briefly** and **succinctly** describe what might happen to a system where $\mathbf{T_I}$ is **decreased**

13. **Properties of a State Transition** (10 Points)
Let's explore some properties of the state transition matrix,**Φ**, for a simple SISO LTI system where $A = \begin{bmatrix} 4 & 0 \\ 7 & 2 \end{bmatrix}$.

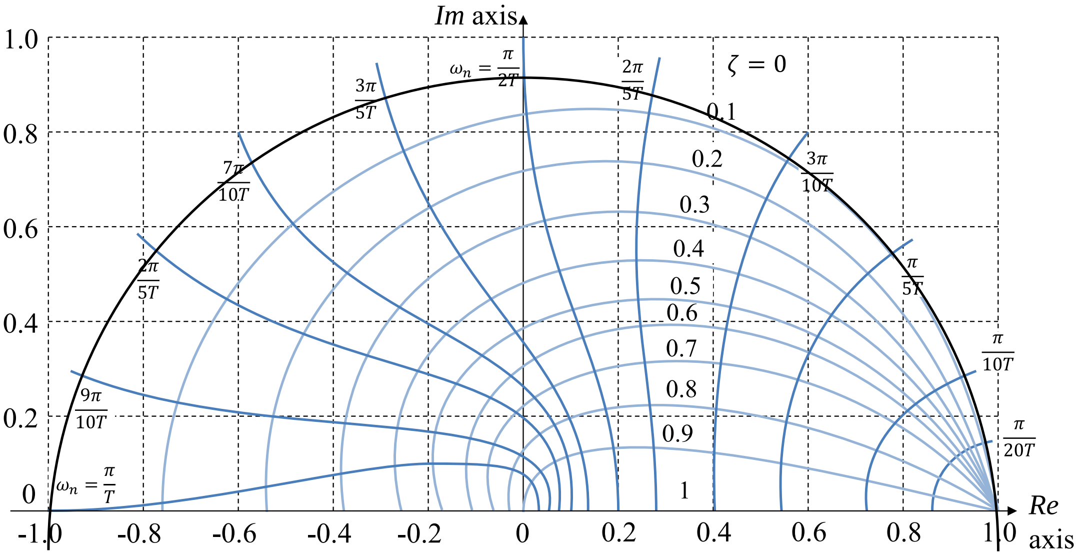
A. **Φ**(t): Please determine **Φ**(t) for this system A.
(**Note:** You may leave it in terms of the matrix exponential)

B. **Φ**(s): Kindly determine: **Φ**(s) for a system A

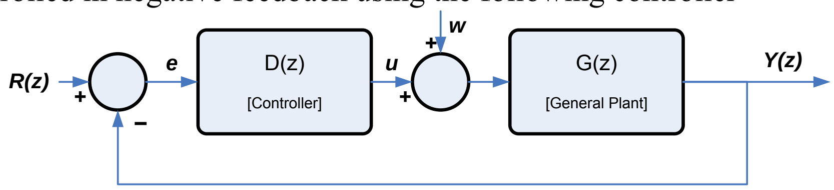
C. Characteristic Polynomial:
Please also find System A's characteristic polynomial.

D. Discrete Representation: Please express this system as a difference equation (i.e. $x(k+1)$ and $y(k)$) assuming a step input at the first step ($u(k)$), ZOH sampling, **H**=**I**, and $\mathbf{\Gamma} = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$.

14. **Racing to Pole Position** (15 Points)
Consider a second order system as described by the loci of roots of constant ζ and ω_n in a normalized $\mathcal{Z}$-plane ($T = 1s$, $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$, $z = e^{Ts}$) as given below.



It is controlled in negative feedback using the following controller



With $D(z) = 13\frac{z-0.9}{z+0.5}$ and $G(z) = 0.05\frac{z+0.96}{(z-1)(z-0.905)}$

A. What is the solution to the closed-loop characteristic equation for this system?

B. What is the damping ratio (ζ) and natural frequency (ω_n) for this case?

C. Now consider the effect of a one-cycle delay in $D(z)$. For this case, what is the new $D_{delayed}(z)$?

D. Will this add any poles/zeros to the closed-loop characteristic equation? **Briefly explain** what difference this delay will make on the system (if any).

15. **First Order Lead/Lag Compensation** (15 Points)

Consider first-order digital compensation for a second-order plant given by:

$$G(s) = \frac{1}{(s+1)(s+10)}$$

It has a compensator given (in the s-domain) by:

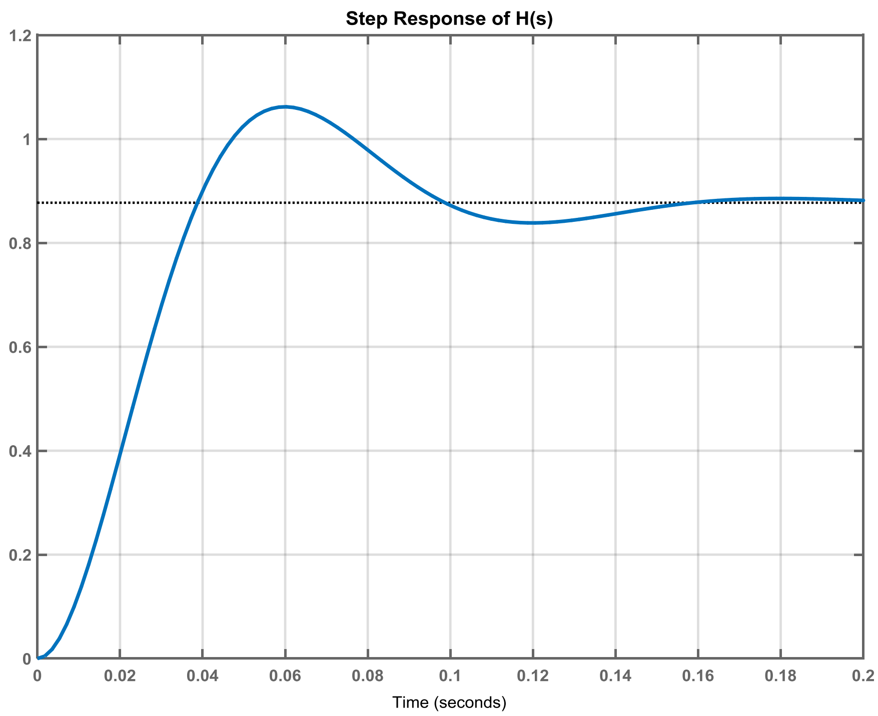
$$C(s) = \frac{3004(s+1)}{s+42}$$

We recall that digital compensator may be thought of a continuous equivalent, given by:

$$D(z) = \frac{K(z-A)}{z-B} = \mathcal{Z}\left\{\frac{k(s+a)}{s+b}\right\}$$

Where $A = e^{-aT}$ and $B = e^{-bT}$ (and $\boldsymbol{K}$ is distinct from $\boldsymbol{k}$). Please assume **50 Hz, ZOH sampling** throughout.

A. Using $D(z)$ above, what is an expression for K when $s = 0$ (i.e. the DC gain)?
B. Using the controller $C(s)$ and the result in (A), what are K, A and B such that we have unity gain at DC?
C. Determine the closed-loop (negative) unity feedback continuous-time transfer function, $H(s)$, for the controller and plant together.
D. Succinctly and very briefly describe a procedure (and/or give pseudocode) to determine $H(z)$.
E. A possible step response for $H(s)$ (i.e. Part C) is given below. For this case, please sketch the step response of $H(z)$.
[**Hint:** Please remember to answer in the **answer booklet**]



END OF EXAMINATION — Thank you !!!

May the LCCODE be with you ☺
(and the wonder ***always*** there)

Table 1: Commonly used Formulae

The Laplace Transform

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt$$

The $\mathcal{Z}$ Transform

$$F(z) = \sum_{n=0}^{\infty} f[n]z^{-n}$$

IIR Filter Pre-warp

$$\omega_a = \frac{2}{\Delta t}\tan\left(\frac{\omega_d \Delta t}{2}\right)$$

Bi-linear Transform

$$s = \frac{2(1-z^{-1})}{\Delta t(1+z^{-1})}$$

FIR Filter Coefficients

$$c_n = \frac{\Delta t}{\pi}\int_0^{\pi/\Delta t} H_d(\omega)\cos(n\omega\Delta t)\,d\omega$$

Table 2: Comparison of Fourier representations.

Time Domain	***Periodic***	***Non-periodic***	
Discrete	***Discrete Fourier Transform*** $\tilde{X}[k] = \frac{1}{N}\sum_{n=0}^{N-1}\tilde{x}[n]e^{-j2\pi kn/N}$ $\tilde{x}[n] = \sum_{k=0}^{N-1}\tilde{X}[k]e^{j2\pi kn/N}$	***Discrete-Time Fourier Transform*** $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)e^{j\omega n}\,d\omega$	***Periodic***
Continuous	***Complex Fourier Series*** $X[k] = \frac{1}{T}\int_{-T/2}^{T/2}\tilde{x}(t)e^{-j2\pi kt/T}\,dt$ $\tilde{x}(t) = \sum_{k=-\infty}^{\infty} X[k]e^{j2\pi kt/T}$	***Fourier Transform*** $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$ $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$	***Non-periodic***
	Discrete	***Continuous***	***Freq. Domain***

Table 3: Selected Fourier, Laplace and z-transform pairs.

Signal	$\longleftrightarrow$	Transform	ROC
$\tilde{x}[n] = \sum_{p=-\infty}^{\infty} \delta[n - pN]$	$\overset{DFT}{\longleftrightarrow}$	$\tilde{X}[k] = \frac{1}{N}$	
$x[n] = \delta[n]$	$\overset{DTFT}{\longleftrightarrow}$	$X(e^{j\omega}) = 1$	
$\tilde{x}(t) = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$\overset{FS}{\longleftrightarrow}$	$X[k] = \frac{1}{T}$	
$\delta_T[t] = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$	
$\cos(\omega_0 t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$	
$\sin(\omega_0 t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = j\pi\delta(\omega + \omega_0) - j\pi\delta(\omega - \omega_0)$	
$x(t) = \begin{cases} 1 & \text{when } \vert t\vert \leqslant T_0, \\ 0 & \text{otherwise.} \end{cases}$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \frac{2\sin(\omega T_0)}{\omega}$	
$x(t) = \frac{1}{\pi t}\sin(\omega_c t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \begin{cases} 1 & \text{when } \vert\omega\vert \leqslant \vert\omega_c\vert, \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = 1$	
$x(t) = \delta(t - t_0)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = e^{-j\omega t_0}$	
$x(t) = u(t)$	$\overset{FT}{\longleftrightarrow}$	$X(j\omega) = \pi\delta(w) + \frac{1}{jw}$	
$x[n] = \frac{\omega_c}{\pi} \operatorname{sinc} \omega_c n$	$\overset{DTFT}{\longleftrightarrow}$	$X(e^{j\omega}) = \begin{cases} 1 & \text{when } \vert\omega\vert < \vert\omega_c\vert, \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = 1$	all s
(unit step) $x(t) = u(t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{1}{s}$	
(unit ramp) $x(t) = t$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{1}{s^2}$	
$x(t) = \sin(s_0 t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{s_0}{(s^2 + {s_0}^2)}$	
$x(t) = \cos(s_0 t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{s}{(s^2 + {s_0}^2)}$	
$x(t) = e^{s_0 t} u(t)$	$\overset{\mathcal{L}}{\longleftrightarrow}$	$X(s) = \frac{1}{s - s_0}$	$\mathfrak{Re}\{s\} > \mathfrak{Re}\{s_0\}$
$x[n] = \delta[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = 1$	all z
$x[n] = \delta[n - m]$	$\overset{z}{\longleftrightarrow}$	$X(z) = z^{-m}$	
$x[n] = u[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{z}{z - 1}$	
$x[n] = z_0^n u[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{1}{1 - z_0 z^{-1}}$	$\vert z\vert > \vert z_0\vert$
$x[n] = -z_0^n u[-n - 1]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{1}{1 - z_0 z^{-1}}$	$\vert z\vert < \vert z_0\vert$
$x[n] = a^n u[n]$	$\overset{z}{\longleftrightarrow}$	$X(z) = \frac{z}{z - a}$	$\vert z\vert < \vert a\vert$

Table 4: Properties of the Discrete-time Fourier Transform.

Property	Time domain	Frequency domain
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\omega}) + bX_2(e^{j\omega})$
Differentiation (frequency)	$nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
Time-shift	$x[n-n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
Frequency-shift	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
Convolution	$x_1[n] * x_2[n]$	$X_1(e^{j\omega})X_2(e^{j\omega})$
Modulation	$x_1[n]x_2[n]$	$\frac{1}{2\pi}X_1(e^{j\omega}) \circledast X_2(e^{j\omega})$
Time-reversal	$x[-n]$	$X(e^{-j\omega})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(e^{j\omega}) = X^*(e^{-j\omega})$
Symmetry (imag)	$\mathfrak{Re}\{x[n]\} = 0$	$X(e^{j\omega}) = -X^*(e^{-j\omega})$
Parseval	$\displaystyle\sum_{n=-\infty}^{\infty} \|x[n]\|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left\|X(e^{j\omega})\right\|^2 d\omega$	

Table 5: Properties of the Fourier series.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1[k] + bX_2[k]$
Differentiation (time)	$\dfrac{d\tilde{x}(t)}{dt}$	$\dfrac{j2\pi k}{T}X[k]$
Time-shift	$\tilde{x}(t-t_0)$	$e^{-j2\pi k t_0/T}X[k]$
Frequency-shift	$e^{j2\pi k_0 t/T}\tilde{x}(t)$	$X[k-k_0]$
Convolution	$\tilde{x}_1(t) \circledast \tilde{x}_2(t)$	$TX_1[k]X_2[k]$
Modulation	$\tilde{x}_1(t)\tilde{x}_2(t)$	$X_1[k] * X_2[k]$
Time-reversal	$\tilde{x}(-t)$	$X[-k]$
Conjugation	$\tilde{x}^*(t)$	$X^*[-k]$
Symmetry (real)	$\mathfrak{Im}\{\tilde{x}(t)\} = 0$	$X[k] = X^*[-k]$
Symmetry (imag)	$\mathfrak{Re}\{\tilde{x}(t)\} = 0$	$X[k] = -X^*[-k]$
Parseval	$\displaystyle\frac{1}{T}\int_{-T/2}^{T/2} \|\tilde{x}(t)\|^2 dt = \sum_{k=-\infty}^{\infty} \|X[k]\|^2$	

Table 6: Properties of the Fourier transform.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$
Duality	$X(jt)$	$2\pi x(-\omega)$
Differentiation	$\frac{dx(t)}{dt}$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{1}{j\omega}X(j\omega) + \pi X(j0)\delta(\omega)$
Time-shift	$x(t-t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency-shift	$e^{j\omega_0 t}x(t)$	$X(j(\omega-\omega_0))$
Convolution	$x_1(t) * x_2(t)$	$X_1(j\omega)X_2(j\omega)$
Modulation	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(j\omega) * X_2(j\omega)$
Time-reversal	$x(-t)$	$X(-j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Symmetry (real)	$\mathfrak{Im}\{x(t)\} = 0$	$X(j\omega) = X^*(-j\omega)$
Symmetry (imag)	$\mathfrak{Re}\{x(t)\} = 0$	$X(j\omega) = -X^*(-j\omega)$
Scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{j\omega}{a}\right)$
Parseval	$\int_{-\infty}^{\infty} \|x(t)\|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \|X(j\omega)\|^2\, d\omega$	

Table 7: Properties of the z-transform.

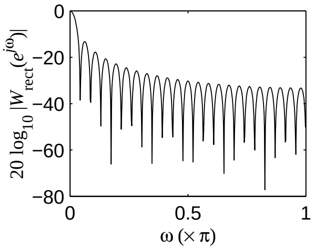
Property	Time domain	z-domain	ROC
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Time-shift	$x[n-n_0]$	$z^{-n_0}X(z)$	$R_x^\dagger$
Scaling in z	$z_0^n x[n]$	$X(z/z_0)$	$\|z_0\|R_x$
Differentiation in z	$nx[n]$	$-z\frac{dX(z)}{dz}$	$R_x^\dagger$
Time-reversal	$x[-n]$	$X(1/z)$	$1/R_x$
Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(z) = X^*(z^*)$	
Symmetry (imag)	$\mathfrak{Re}\{X[n]\} = 0$	$X(z) = -X^*(z^*)$	
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Initial value	$x[n] = 0, n < 0 \Rightarrow x[0] = \lim_{z\to\infty} X(z)$		

† $z = 0$ or $z = \infty$ may have been added or removed from the ROC.

Table 8: Commonly used window functions.

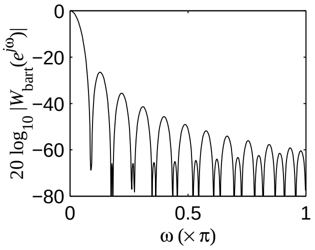
Rectangular:

$$w_{\text{rect}}[n] = \begin{cases} 1 & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



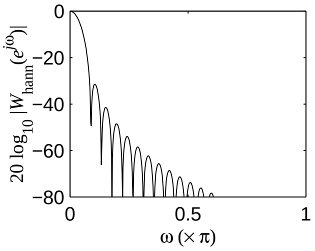
Bartlett (triangular):

$$w_{\text{bart}}[n] = \begin{cases} 2n/M & \text{when } 0 \leqslant n \leqslant M/2, \\ 2 - 2n/M & \text{when } M/2 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



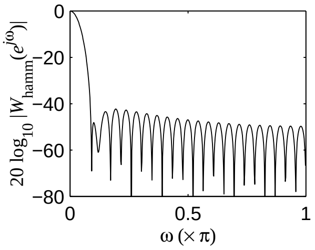
Hanning:

$$w_{\text{hann}}[n] = \begin{cases} \frac{1}{2} - \frac{1}{2}\cos\left(2\pi n/M\right) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



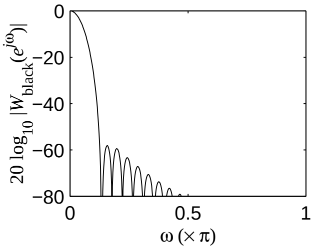
Hamming:

$$w_{\text{hamm}}[n] = \begin{cases} 0.54 - 0.46\cos\left(2\pi n/M\right) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



Blackman:

$$w_{\text{black}}[n] = \begin{cases} 0.42 - 0.5\cos\left(2\pi n/M\right) \\ \quad + 0.08\cos\left(4\pi n/M\right) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



Type of Window	Peak Side-Lobe Amplitude (Relative; dB)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)
Rectangular	-13	$4\pi/(M+1)$	-21
Bartlett	-25	$8\pi/M$	-25
Hanning	-31	$8\pi/M$	-44
Hamming	-41	$8\pi/M$	-53
Blackman	-57	$12\pi/M$	-74