

ELEC 3004 – Systems: Signals & Controls

Assignment 0: Answer Key

Overview

This is simply an answer key to the sample problems posted. Given that this “assignment” was primarily for the purpose of background review and for getting an introduction to the Platypus system, the questions were not meant to be hard or complicated.

Question 1:

The Laplace transform of $\mathbf{Delta}(t)$ is 1

The Laplace transform of the step function $\mathbf{u}(t)$ is $\frac{1}{s}$

Question 2:

$$\frac{2s + 4}{s^2 + 4s + 3} = \frac{2(s + 2)}{(s + 3)(s + 1)} = \frac{1}{s + 3} + \frac{1}{s + 1}$$

The poles are the solution to the Laplacian in the denominator and are -1,-3.
The zeros are the solution to the Laplacian in the numerator and is -2.

Question 3:

Capacitance is given by:

$$C = \frac{q}{V}$$

Taking the derivative with respect to time gives

$$C(t) = \frac{\frac{dq(t)}{dt}}{\frac{dV(t)}{dt}} = \frac{I(t)}{\frac{dV}{dt}}$$

$$I(t) = C(t) \frac{dV}{dt}$$

Thus,

$$V(t) = \int_0^t \frac{1}{C(t)} I(\tau) d\tau$$

Also, the circuit is essentially a voltage divider. Recalling the “impedance” of a constant capacitor as $Z = \frac{1}{Cs}$ and applying Ohm’s law gives:

$$V(s) = IZ = I \left(\frac{1}{Cs} \right) = I \left(\frac{1}{C} \right) \left(\frac{1}{s} \right)$$
$$\rightarrow V(t) = \left(\frac{1}{C} \right) \int_0^t I(\tau) d\tau$$

Which is the same as above for the case of constant capacitance.