Lecture: Open Lecture I

→ Linearity: A system that admits superposition & allows for linear analysis.

1) Additivity
   - Given an input $x_1(t)$, the system produces an output $y_1(t)$, and given a second input $x_2(t)$, it produces an output $y_2(t)$.
   - Then, the system has additivity if $x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$ for any arbitrary $x_1(t) + x_2(t)$.

2) Homogeneity & Scaling
   - Given: $x(t) \Rightarrow y(t)$
   - Then a linear, scaled $a \cdot x(t)$ must give $a \cdot y(t)$.

Superposition
   - Given: $x_1(t) \Rightarrow y_1(t)$
     $x_2(t) \Rightarrow y_2(t)$

   $x(t) = a \cdot x_1(t) + b \cdot x_2(t)$

   $y(t) = a \cdot y_1(t) + b \cdot y_2(t)$
Graphically:

\[ x(t) \rightarrow \text{Linear System} \rightarrow w(t) \rightarrow y(t) \rightarrow y_0(t) + \]

- **TIME-VARYING or TIME-INVARIANT**

- **Systems w/ Memory**

A system is said to have memory if the output \( y(t) \) at an arbitrary time \( t \) depends on an input value \( x(t_0) \) other than \( \) or in addition to \( x(t_0) \).

The values may be there in the past \( t < t_0 \) or the future \( t > t_0 \).

- **Continuous / Discrete Time**

\[ \sin (wt) \rightarrow \sin [kT] \rightarrow R \rightarrow Z \]
Invertable System

A system is invertable if we can determine the input by observing the output

\[ y(t+5) = x^3(t) \]
\[ y(t) = 2x(t+1) + 3 \]

Non-invertable System:

\[ y(t) = \sin[x(t)] \]
\[ y(t) = x^2(t) \]

Stable:

is one which admits BIBO

- Bounded input \| Bounded-output

Stable: \[ |x(t+5)| \leq B_1 \]
\[ \Rightarrow \quad |y(t+1)| \leq B_2 \]