ELEC 3004 / 7312 – Digital Linear Systems: Signals & Controls **2014**

Problem Set 4: Controls

Total marks: 80 **Due Date:** Sunday, June 8, 2014 (at 11:59pm, AEST)

Note: The assignment consists of seven questions. For Q1-Q3, please answer 2 of the 3 questions. That is, the assignment paper consists of 90 marks worth of questions. Please complete just 80 marks worth of questions. No extra credit will be given for answering all three questions.

This assignment is worth **15%** of the final course mark. Please submit answers via Platypus. It is requested that solutions, including equations, should be typed please. The final grade is the median of the marks from the peer reviews and the staff (with provisions for review). Finally, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions. Thank you very much. :--)

FOR QUESTIONS Q1, Q2, and Q3 ONLY --
PLEASE ANSWER **TWO** of the **THREE** questions of your choosing

Q1. An E-Z Sine [10 points]

Show that the z-Transform of $f(t) = sin(\omega t)$ is

$$F(z) = \frac{z sin(\omega T)}{z^2 - 2z cos(\omega T) + 1}$$

Q2. An E-Z Step [10 points]

Show that the z-Transform of the unit step function $u(t)$ is

$$U(z) = \frac{z}{z-1}$$

Hints:

- $u(t)$ is not to be confused with the control signal vector **u(t)**
- Recall that the infinite geometric series may be written as $(1 - bx)^{-1}$

Q3. An E-Z Curve [10 points]

(Based on Franklin, Powell, and Workman, Problem 5.1)

Use the $z = e^{sT}$ mapping function and prove that the curve of constant ζ in the s-plane is a logarithmic spiral in the z-plane.

Q4. S and Z: Best Mates or Mirror Images? [20 points]

Consider the system defined by the differential equation below.

$$x'' + 4x' + 8x = 0$$

The *Forward Rule* (stated below) between the z and s domains implies the following substitution.

$$s \leftarrow \frac{z-1}{T}$$

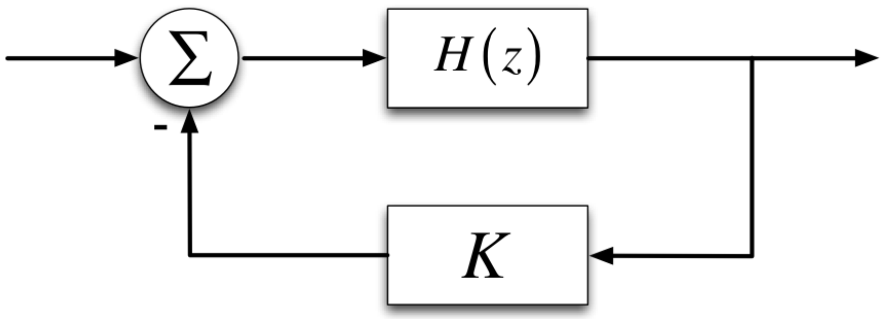
For this question, unless otherwise stated, **please provide an algebraic solution**.

1. Directly (i.e., without using any kind of transfer function) discretize this differential equation. Use the *Forward Rule* (in the time domain).

2. Derive a transfer function $H(z)$ from the difference equation derived in part 1.

3. Will $H(z)$ be stable for any value of T?
   Derive a range of stable T values, or prove that none exists. In either case, explanations should refer to the z plane.

4. For which values of K will the system below be stable?
   Use MATLAB and answer this question graphically, with a brief explanation and reference to the z plane.

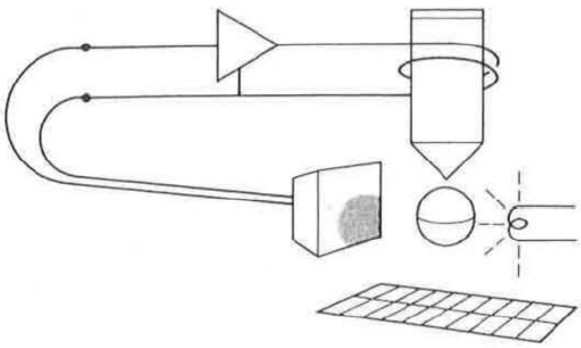


5. Starting from $H(s)$ of the original differential equation, discretize the system using *Tustin's Method*.
   Check your discretization using MATLAB [hint: `c2d`].
   Has this method provided a stable discretization?

Q5. Suspension of Disbelief **[20 points]**

(Based on Franklin, Powell, and Workman, Problem 5.9)

Consider a magnetic mass suspended by means of an electromagnet whose current is controlled by the position of the mass which is sensed using a position sensor



For an electromagnet force of position and current given by f(x,I), the dynamic of the mass is given by:

$$m\frac{d^2x}{dt^2} = -mg + f(x,I)$$

At equilibrium, the magnet force balances the gravity force. Let's define $\mathbf{I_0}$ as the equilibrium current. If we write $I = I_0 + i$ and expand $\boldsymbol{f}$ about x = 0 and I = $\mathbf{I_0}$, and if we neglect higher-order terms, we obtain

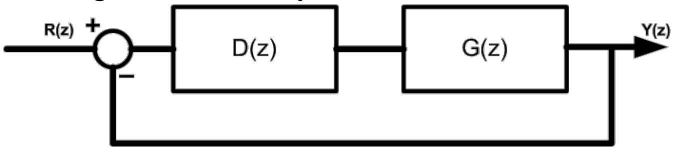
$$m\frac{d^2x}{dt^2} = k_1x + k_2i$$

Reasonable values are **m=10 g**, $\mathbf{k_1}$**=20 N/m**, $\mathbf{k_2}$**=0.5 N/A**.

a. Compute the transfer function for ***i*** (current in magnet/solenoid) to **x** (position of the suspended magnetic mass).
b. Let the sample period be **0.01 sec** and compute the plant discrete transfer function when used with a zero-order hold
c. Design a digital control for the magnetic levitation to meet the specifications: $\mathbf{t_r \le 0.1}$ **sec**, $\mathbf{t_s \le 0.5}$ **sec**, and **overshoot ≤ 10%**.

Q6. Driving a Hard Drive [10 points]

Consider a digital controller for a disk drive system in which the disk spins with a constant speed and the has a time between readings of T. This may be modeled as:



Where $G(s) = [ZOH]\left[\frac{5}{s(s+20)}\right] = \left[\frac{1-e^{-sT}}{s}\right]\left[\frac{5}{s(s+20)}\right]$

a. Determine $G(z)$ [please state your assumptions (e.g., ZOH)]

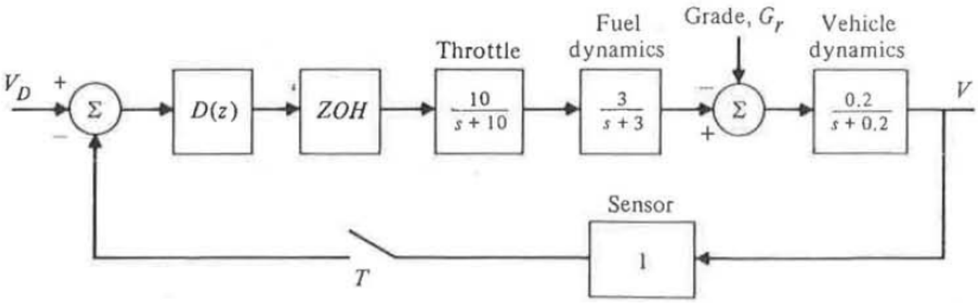
b. For $T = 1ms$, a proportional controller ($D(z) = K$), what is the open loop transfer function?
[**Hint:** What happens to G(s) (or how can G(s) be simplified) if the fast pole ($s = -20$) is neglected?]

c. What is the closed-loop transfer function?
[**Hint:** Apply the same assumptions as was done in Part b]

Q7. Cruising to the End (Driving a Cruise Control) [10 points]

(Based on Franklin, Powell, and Workman, Problem 5.12)

Consider an automotive cruise control as described below:



a. Design a PD controller to achieve a $\mathbf{t_r}$ of 5 sec (on 0% grade)

b. Determine the speed error on a 10% grade (i.e., 5.7° or G_r=5.7)

c. Briefly discuss what would be the best control linear architecture (i.e, P, PD, PI, PID, etc.) to adopt for the roads around the Brisbane metro region.
[**note:** this includes regions such as Mt. Coot-tha and the QPS is short of revenue]