ELEC 3004 / 7312 – Digital Linear Systems: Signals & Controls **2014**

Problem Set 2: Linear Systems

Total marks: 80 **Due Date:** Sunday, April 27, 2014 (at 11:59pm, AEST)

Note: This assignment is worth **15%** of the final course mark. Please submit answers via Platypus. It is requested that solutions, including equations, should be typed please. The final grade is the median of the marks from the peer reviews and the staff (with provisions for review). Finally, the tutors will ***not*** assist you further unless there is real evidence you have attempted the questions. Thank you very much. :--)

Short Questions

(Please keep it simple)

Q1. Carrier Waves **[5 points]**

In class it was said that a carrier signal is usually a sinusoid. When might it be good to use non-sinusoidal symmetrical carrier waves?

Q2. Shattering Past Nyquist? **[5 points]**

A crystal glass has a resonant frequency of 1100 Hz. A soprano sings this note exactly. If the note (treat it as a pure tone or sine wave) is amplified and transmitted using a digital amplifier that samples at 2000 Hz, will the glass shatter on the other side (assume arbitrarily high amplifier power)? **(Please briefly justify)**

Q3. The Nonuniqueness of Discrete-Time Sinusoids **[10 points]**

A. Prove that a discrete-time sinusoid $f[k] = \cos(\omega kT) = \cos(\Omega k)$ is not unique.
B. Using the findings of part A, select two indistinguishable discrete cosines of different frequencies. Then plot them In MATLAB using *stem()* to demonstrate the principle of part A.

Q4. Convolution Theorem in MATLAB **[10 points]**

A. Given two vectors

A = [1, 7, 4, 2, 9, 1]

B = [6, 4, 2, 1, 1, 9]

Use MATLAB to convolve A and B. Plot A, B and their convolution using the stem() function.

B. Using the *Convolution Theorem*, recalculate part A using the appropriate Fourier method. Do you get the result you expect?

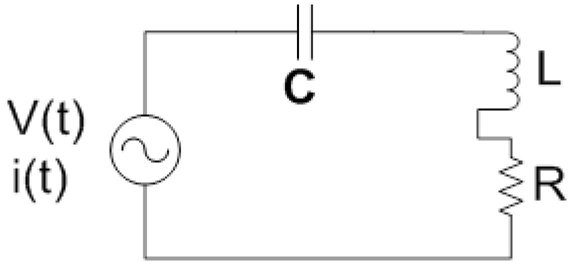
Hint: Look up MATLAB's conv() and fft().

Long Questions

(Please be sure to carefully justify your answers)

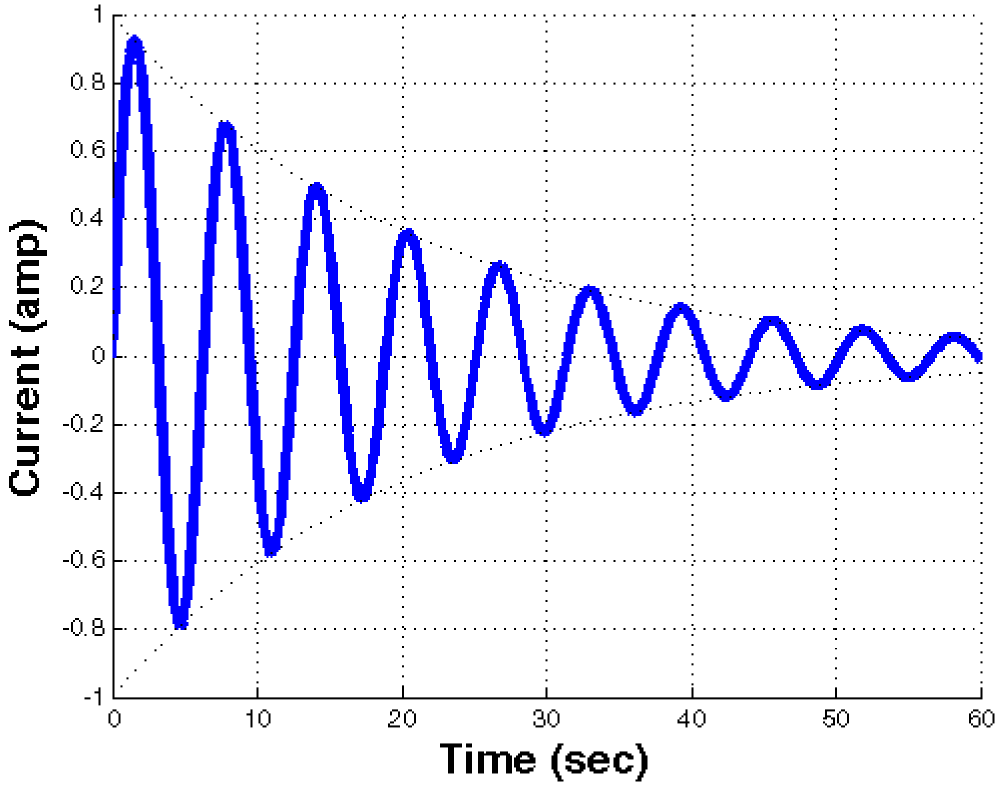
Q5. Second Order Systems **[10 points]**

Consider the following series RLC circuit system.



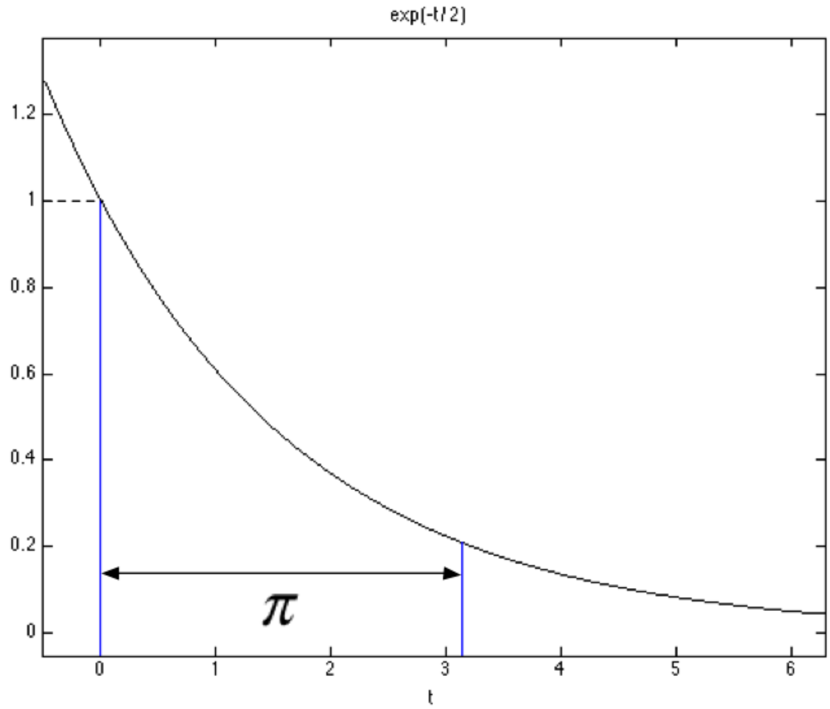
Recall that for such a circuit, the response is second order and that
$\omega_0 = \frac{1}{\sqrt{LC}}$. $\alpha = \frac{R}{2L}$, and $\zeta = \frac{\alpha}{\omega_0}$

Determine the C for a circuit with R=1Ω, L=10H and the following impulse response:
(hint: the values might be larger than "common" parts)



Q6. Fourier Series: The Next Episode **[10 points]**

Find the trigonometric Fourier series for the exponential $e^{-\frac{t}{2}}$, over the interval $0 \le t \le \pi$.



Hint: You are required to represent $f(t)$ only over the interval $0 \le t \le \pi$. This allows you to set the fundamental period to $T_0 = \pi$. What does this tell you about the fundamental frequency of this Fourier Series?

Q7. Hi-Fi Lectures **[15 Points]**

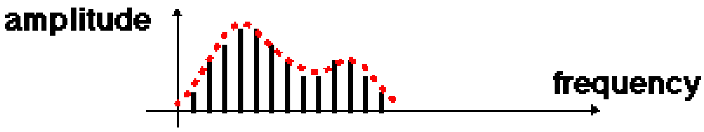


Figure 1

Consider an audio signal x(t) with spectrum as given in Figure 1. Assume the audio signal contains primarily human speech with a bandwidth of B=20Khz. A 44.1kHz sampler will store 44100 samples per second of audio. A lecture recording of 110 minutes will require ~4.85x10^6 samples.

An engineer decides this is too much and uses a 10 kHz sampler. This so called engineer from the real world, graduating from an unmentionable university, recalls that through Nyquist's theorem this will allow him to represent frequencies from -5kHz to 5kHz. Since most of human speech falls in this region anyway, he calculates that there shouldn't be a significant loss of audio quality.

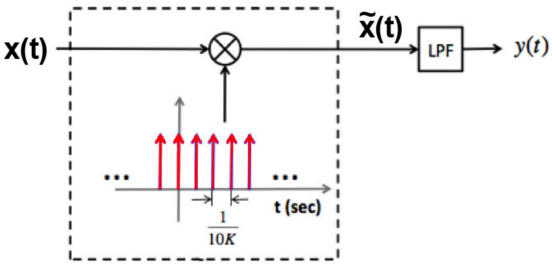


Figure 2

The engineer builds the above system and expects that everything beyond 5Khz will be simply filtered by the low pass filter. Much to his surprise, the output didn't resemble his voice at all and contained strange noise. Besides graduating from the wrong university, what did the engineer do wrong?

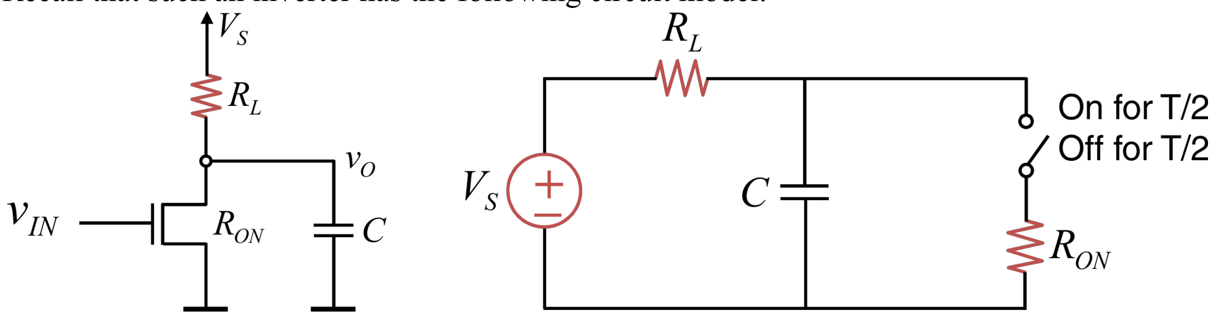
Being a UQ graduate, you have been employed to fix this mess. Given that you cannot change the sampling frequency, how would you fix this?

Q8. Hot Chips **[15 Points]**

The conduction of heat is given by Fourier's Law, a first-order linear differential model. With the thermal energy heat transfer rate, q_x (typically measured in Watts), given by:

$$q_x = -kA\frac{dT}{dx}, \quad q_x'' = \frac{q_x}{A}$$

Lets consider this for the application of heat transfer from a chip. Chiefly, Prof von Complementary is interested in transferring heat from an integrated set of n-channel MOSFET inverter logic circuits. Recall that such an inverter has the following circuit model:



The power of which is given by:

$$\overline{P} = \frac{V_S^2}{2R_L} + CV_S^2 f$$

Where: V_s is the supply voltage, R_L is a pull-up resistor, C is the capacitance and f is the switching frequency. For speed, we set f=3GHz. The IC consists of 5 million of these circuits (gates). Assume C=1 femtoF, R_L=10 kΩ, and V_s=5V (TTL), N=5 million (as above).

(1) How much power is lost by this IC?

(2) If the surface area of the chip is 1 mm^2, what is the heat flux (q'' or thermal power/area) of this circuit? Comment on this nature of this value.
[hint: What else might have a similar q''? What might happen to this circuit?]

(3) To dissipate the heat, Prof von Complementary decides to add a heatsink (so as to increase the area and reduce the heat flux). Using the equation above, will the temperature on the ambient side of the heatsink be the same as that on the chip side? (Assume $k_{Aluminum}$=237 W/mK, A=5×10^{-6} m^2)
. If the equivalent thermal resistance of the heat sink is 36.9 K/W and ambient temperature is 20°C, what is the temperature of the chip surface (assuming the heat flux from part (2)).

(4) What are **two** specific design changes that Prof von Complementary can do to reduce the temperature of the surface of the inverter array?
[*hint*: deeply consider where the power is going and the overall design here. Also reflect on what the current IC industry does and what values they might use. Might heat transfer might actually be the limiting factor in IC performance?]