ELEC 3004/7312: Signals Systems & Controls I
Assignment 1 (Revised Twice*), Due: 23rd March, 2012

Note: This assignment is worth **10%** of the final course mark. You should spend approximately 3 hours preparing for the tutorial. The tutors will *not* assist you further unless there is real evidence you have attempted questions prior to the tutorial. Beyond the lecture and tutorial sessions, it is estimated that you will need 4 to 5 hours to complete the assignment (**7-8 hours total**).

Total marks: 100

1. **[20]** Even and odd signals refer to their symmetry under time reversal.
An even signal is defined such that it is identical to its time-reverse counterpart.
Consider a sinusoidal source $x(t) = \sin(2\pi ft + \phi)$, configured as a 10Hz band-limited source such that $x(t) = \cos(20\pi t)$. The signal **w[n]** is sampled from **x(t)** at 100 Hz for 10 seconds (for a time vector **t[n]** and a corresponding index vector **n[t]**, written together as **w[n[t]]** or **w[t]**):
   a) Is it possible to determine if **w[t]** is even or not? Why?
   If so, please describe a simple algorithm for determining if it is even or not
   If not, please describe what changes (to constraints, assumptions, or parameters, initial condition, etc.) could be made so as to make this possible.
   b) If the frequency, *f*, of the source signal **x(t)** was to be increased, what is the maximum bandwidth (frequency) of **x(t)** for which it is possible to determine if the signal **x(t)** is even?

   Extra Credit: What would it be for **w[t]**? Consider the aliasing and the phase – what happens if $f_s/2<f<f_s$? What happens if $f_s<f<2f_s$? (Remember the stroboscopic effect?)

2. **[21]** We may describe a system as being:
   - linear[1]
   - time-invariant[2]
   - causal[3]

   Make a 3x7 Matrix in which you determine which of these three properties hold (yes/no) for:
   a) Backward differencer (differentiator): $y[t] = x[t] - x[t-1]$
   b) Forward differencer: $y[t] = x[t+1] - x[t]$
   c) Central differencer: $y(t) = x(t+\tfrac{1}{2}) - x(t-\tfrac{1}{2})$
   d) Multiplier: $y[n] = Gx[n]$
   e) Integrator or Accumulator : $y[n] = \sum_{1}^{n+1} x[k]$
   f) Compressor: $y[n] = x[Mn]$, where M is a positive integer
   g) Square-Law: $y(t) = x^2(t)$

3. **[19]** Recall that $Ge^{(a+ib)} = G(e^a)(e^{ib}) = G(e^a)(\cos(b) + i\sin(b))$. **Why?**

   Hint: one way, of many, is to consider the Taylor series (about 0) for e^x, sin(x), and cos(x):
   $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$, $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$, $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$
   Also, Wikipedia [http://en.wikipedia.org/wiki/Euler%27s_formula] and Wolfram MathWorld [http://mathworld.wolfram.com/EulerFormula.html] have good articles on this.

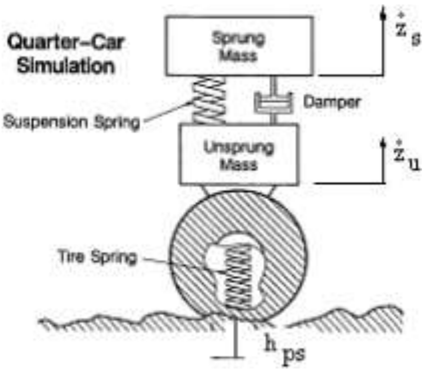
Continued on Page 2 …

[1] Superposition holds, that is for the input/output pairs **x_1[n]** → **y_1[n]** and **x_2[n]** → **y_2[n]**, the combination **ax_1[n] + bx_2[n]** → **ay_1[n] + by_2[n]**
[2] Characteristics of the system are fixed over time, so for a system with input **x[n]** and output **y[n]**, the input **x[n-m]** will give the output **y[n-m]**
[3] The output only depends on time inputs at the present and past times. The system is nonanticipative.

4. [40] Potholes and the Quarter-Car Model

It is desired to measure a pothole from an accelerometer in a car.

Recall that a car's suspension can be viewed as a spring (for the tyre), a mass for the wheel (called the "unsprung mass"), the suspension spring and damper and mass for the car (typically ¼ the vehicle's mass). This is called the quarter-car model. Also recall that an accelerometer is itself a mass connected to a (very stiff) spring that is connected to the outer case (called "ground", but not to be confused with Earth ground or the vehicle chassis).



Quarter-Car Model (from Google Images c/o US DOT http://goo.gl/wOY5m)

a) What is the overall order of the system as seen from the accelerometer (proof mass or case, though please specify) with respect the pothole (assume the ground is infinitely stiff).
b) Sketch (or plot) the expected signal for running over a pothole. Consider the pothole as an impulse, the car's damper as overdamped (shocks are new), and accelerometer as over-damped too (sealed in gas).
c) Sketch (or plot) the expected signal for dropping an accelerometer on to the floor of the car (presuming, of course, that it is not placed on the floor to begin with and, for example, was placed on top of the dash) with the car not driving over a pothole. Where needed, assume the same as in (b). Is this signal the same as in (b)?
d) What sampling rate would you recommend to use (assume an average passenger car, such as a Toyota Corolla)? Consider that excessively high sampling rates use more energy (thus, shorter on-battery operations), give more data to process, and risk more noise. Briefly and succinctly justify your answer based on your research.

Note: For curiosity, lookup the Street Bump Android App – it is being trailed for finding potholes in Boston.

Extra Credit: For a system like this (e.g., Street Bump), how might you distinguish the signal from an bump from that of a collision?

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[*] Revised March 16, 2012:

- Removed "turn in…" reminder. It should be submitted via the submission system please.
- Question 1: x(t) corrected to include time (from cos(20π) to cos(20π*t)). w[t] to w[n[t]]
- Question 2: Added footnote to clarify meanings of terms
- Question 3: Font error with Sigma fixed (okay in PDF not on class printout)
- Question 4: Typo (an to a), 4(c) "phone" to accelerometer, Extra credit question added

Revised March 17, 2012:

- Question 1: w[t] expanded as w[n[t]] (they are the same). (b) Remove reminder about x(t) and replaced with extra credit for also solving this for w[t]
- Question 4: (a) Added the option to consider or neglect the accelerometer's dynamics