1. Linearity
Is the system described by the following linear or non-linear? (Please briefly justify your answer)

\[ y(t) = \frac{dx(t)}{dt} x(t) + x(t) + 1 \]

Non-linear

The first term involves a multiplication of \( x, x \), which is a non-inear term to see this. Consider

2. Convolution
Recall that convolution is given by:

\[ x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)x_2(t-\tau)d\tau \]

What are the following convolutions?

(a) \( x(t) * \delta(t) \)

\[ x(t) \quad \text{(by definition)} \]

(b) \( x(t) * x(t) \)

\[ t \cdot x(t) \]

Recall that convolution in the time domain is a multiplication in the Laplace domain. So \( \mathcal{L}\{x(t)^2\} \).
3. Second Order Systems

Recall that for a series RLC circuit, the response is second order and that \( \omega_0 = \frac{1}{\sqrt{LC}} \), \( \alpha = \frac{R}{2L} \), and \( \zeta = \frac{\alpha}{\omega_0} \).

Determine the \( C \) for a circuit with \( R=1\Omega, L=1H \) and the following response:

(hint: the values might be larger than “common” parts, \( \frac{\sqrt{3}}{2} = 0.866 \))

- \( T = 7.3 \text{ sec} \) \( \Rightarrow \) \( \frac{1}{T} = \frac{1}{7.3} \) \( \Rightarrow \) \( \omega_I = 2\pi f_D = \frac{2\pi}{7.3} \)

- \( \omega_I = \omega_0 \sqrt{1-\zeta^2} \)

\[
\omega_I = \left(\frac{1}{\sqrt{LC}}\right) \sqrt{1-\left(\frac{\alpha}{\omega_0}\right)^2} \\
= \left(\frac{1}{\sqrt{LC}}\right) \sqrt{1-\left(\frac{1}{2}\right)^2} \\
= \frac{1}{\sqrt{LC}} \sqrt{1-\frac{1}{4}} \\
= \frac{1}{\sqrt{\frac{1}{2}L}} \sqrt{\frac{3}{4}} \\
= \frac{\sqrt{3}}{2} \\
\Rightarrow \zeta = \frac{\sqrt{3}}{2} \\
\Rightarrow \zeta^2 = \frac{3}{4}
\]

- \( 3 = \frac{L}{2L} \cdot \frac{\sqrt{3}}{1} = \frac{\sqrt{3}}{2} \)

\( \Rightarrow 3 = \frac{\sqrt{3}}{2} \)

(using values from above)

\( \Rightarrow 3^2 = \frac{c}{4} \)

\( \Rightarrow c = 12 \)

Note: The period is read as \( \frac{2\pi}{T} = \frac{2\pi}{14} \)

Then \( T = 6.28 \text{ sec} \) \( \Rightarrow \omega_D = 1 \)

\( \Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{1}{L} \)

\( \Rightarrow \left(\frac{2\pi}{7.3}\right)^2 = \frac{1}{L} \)

\( \Rightarrow \frac{c}{L} = \frac{1}{22} \)