Measure of a Signal

Signal Size

\[ s = \int_0^T |u(t)| \, dt \]

- \( |u(t)| \) is just some arbitrary "signal"
- ought integrates to zero
- 1. Absolute value
- 2. Magnitude \( \Rightarrow ||u(t)||^2 \)

Signal Energy

\[ E = \int_{-\infty}^{\infty} u^2(t) \, dt \]

- But what if \( u(t) \) is not bounded — for example a sine wave —
- Then \( \infty \) energy

Signal Power

\[ p = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} u^2(t) \, dt \]

- \( p \): the time average mean of \((\text{amplitude})^2\)
- \( \sqrt{p} \): RMS (Root Mean Square) value of the amplitude

- "Power" is not "power" in a typical energy sense (not in Watts)

- Infinite signals can have finite power
- ex.: unit step (which is not periodic or statistically regular)
  class have finite power
Signal Classifications

1. Continuous - Time

\[ u(t) \]

In continuous time: \( t \) is of a continuous set \( t \in \mathbb{R} \) (reals)

or any other continuous set such as \( t \in \mathbb{R} \times \mathbb{R} \), etc.

2. Discrete - Time

\[ u[n] \]

Relationship to continuous time: \( u[n] = u(nT) \)

\( t = \mathbb{Z} \) (integers)

\( t_n = \{ t_0, t_1, t_2, \ldots, tn \} \)
3. Analog & Digital Signals

- **Continuous** vs. **Discrete** on the y-axis

- **Analog**

- **Digital**

- Digital is much better for communications
  - It is noise robust
  - Small changes in values due to noise don't change the signal

4. Real & Complex Signals

- \( x(t) = x_1(t) + jx_2(t) \)

- \( x(t)^* = x_1(t) - jx_2(t) \)

\[ \Rightarrow \text{consider} \quad s = x_1 + x_2 \quad j \quad \] (if \( x_1 = 0 \) and \( x_2 \) is real, then this is the familiar \( s = \sigma + j\omega \))

\[ \Rightarrow e^{st} = e^{(x_1 + x_2)j} = e^{x_1t} \cdot e^{x_2tj} \]

\[ = e^{x_1t} \left( \cos(x_2t) + j \sin(x_2t) \right) \]
5) Deterministic & Random

Deterministic are ones whose description is known completely

\[ \sin(t) \]

There may be complex (e.g. non-linear)

Random

Those signals whose description is known incompletely via statistical or probabilistic descriptors

\[ \exists \phi: \sin(wt + \phi) \] where \( \phi \) is unknown

\[ \text{White Noise} \]

Avg. is that of a \( \text{Sin} \)

(A zero-meaned, Gaussian of var 0) \[ \mathcal{N}(0, \sigma) \]
(0) Constant

\[ u(t-t_0) = \begin{cases} A & t > t_0 \\ 0 & t < t_0 \end{cases} \]

If \( A = 1 \) \( \Rightarrow \) Heaviside function

(1) Unit Step

(2) Unit Impulse

Dirac Delta function

\[ \delta(t) = \begin{cases} 0 & t \neq t_0 \\ \infty & t = t_0 \end{cases} \]
Sinusoidal Signal

\[ x(t) = A \cos (\omega_0 t + \theta) \]

- \( A \): Amplitude
- \( \omega_0 \): Frequency \((\omega_0 = 2\pi f)\)
- \( \theta \): Phase angle

\( A, \omega_0 \): Self-explanatory (mostly)

\( \theta \): Phase angle

We define phase relative to y-axis crossing:

\[ x(0) = A \cos (\theta) \]
\[ \theta = \arccos \left( \frac{A}{x(0)} \right) \]

Period:

\[ T_0 = \frac{2\pi}{\omega_0} \]

(Which intuitively: \( \text{period} \times \text{freq.} = \text{time} \))
Note that the sinusoidal is related to the exponential via Euler's formula

$$e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t} e^{j\omega t} = e^{\sigma t} \left[ \cos(\omega t) + j \sin(\omega t) \right]$$

There are four cases for this:

1. \( s = 0 \) \( \Rightarrow \) A constant \( \text{(trivial)} \)
2. \( s = \sigma, \omega = 0 \) \( \Rightarrow \) A monotonically exponential
3. \( s = \pm j\omega, \sigma = 0 \) \( \Rightarrow \) A sinusoidal
4. \( s = \sigma \pm j\omega \) \( \Rightarrow \) An essentially varying sinusoid
Even & Odd Functions

**Definition**

**Even functions**

\[ f_e(t) = f_e(-t) \]

**Odd functions**

\[ f_o(t) = -f_o(-t) \]

**So what?** (Some properties)

**Even**

- Has the same value at the instant \( t \) and \( -t \)
- Symmetric about the vertical axis

**Odd**

- Its value at instant \( t \) is the negative of its value at the instant \( -t \)
- \( f_{\text{odd}}(t) \) is anti-symmetric about the vertical axis
  (Symmetric about the line \( y = -x \))

**Area**

\[
\int_{-a}^{a} f_e(t) \, dt = 2 \int_{0}^{a} f_e(t) \, dt
\]

\[
\int_{-a}^{a} f_o(t) \, dt = 0
\]

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- Every signal can be expressed as a sum of even and odd components

\[
f(t) = \frac{1}{2} \left[ f(0) + f(-t) \right] + \frac{1}{2} \left[ f(t) - f(-t) \right]
\]

**Even**

**Odd**