Sort Pattern **X**

THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This paper must not be removed from the venue

School of Information Technology and Electrical Engineering
Quiz

METR4202: ROBOTICS & AUTOMATION

August 27, 2018

First Name: ______________________ **Last Name:** ______________________________

Student Number: ______________________________________

Q	Mark
1	
2	
3	
4	
5	

Total __________

Examination Duration: 45 minutes

Reading Time: 10 minutes

Exam Conditions:
This is a Closed Book Quiz

Electronic Materials Permitted In The Exam Venue:
- Calculator – Allowed
- One A4 (Single-Sided) Formula Sheet – Allowed

Instructions To Students:
- Please be sure to place your name and number on **ALL** pages
- Please answer **ALL** questions.
- ⇨ **ALL** Answers **MUST Be Justified** ⇦
  (answers alone are not sufficient)

Thank you! ☺

This quiz consists of Short Answer, Worked Problems, and Multiple Choice. **Please Answer <u>All</u> Questions** below on the quiz paper. Answers must be neat and clear. All answers (except for multiple choice) **must provide a brief justification**. You may use the back of each sheet as scratch paper if needed. The total quiz is worth 100 points.

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1. Putting Transformations Matrices into Perspective [5 Points]

To get started, let's consider the humble process of transforming frames in 3D-space using a homogeneous transformation matrix.

A transformation matrix naturally partitions into four (4) submatrices. Assuming **Rigid Body, Euclidian Space**, please specify the main **sub-components** ***and*** **their matrix dimensions**? (i.e., for the Transformation Matrix for a Euclidian Group)

$$\mathbf{T} = \begin{matrix} \boxed{}_{(\square \times \square)} & \boxed{}_{(\square \times \square)} \\ \boxed{}_{(\square \times \square)} & \boxed{}_{(\square \times \square)} \end{matrix}$$

2. Trigonometry Comes From Linear Algebra [15 Points]

To get started, let's consider two rotation matrices, $[\boldsymbol{A}]$ and $[\boldsymbol{B}]$, that are about the same origin and both rotate a vector about a 3D frame's $\boldsymbol{z}$-axis by the same angle $\boldsymbol{\theta}$.

(a) What are $[\boldsymbol{A}]$ and $[\boldsymbol{B}]$?
(i.e., please expand the matrices, and give their contents/elements inside)?

(b) It intuits that the composite rotation should be $2\boldsymbol{\theta}$.
Please show that $[\boldsymbol{A}][\boldsymbol{B}]$ gives a rotation of $2\boldsymbol{\theta}$**.**

3. Easy as $^{0}1\ldots$ $^{1}2\ldots$ $^{2}3!$ [40 Points]

Consider a planar 3R robot arm with lengths l_1, l_2 and l_3 and frames {0}, {1}, {2} and {3} at the joints as shown below in Figure 1. The angles between the frames are given by θ_1, θ_2 and θ_3 [i.e. with $\theta_1 = {}^{0}\theta_1, \theta_2 = {}^{1}\theta_2, \theta_3 = {}^{2}\theta_3$].

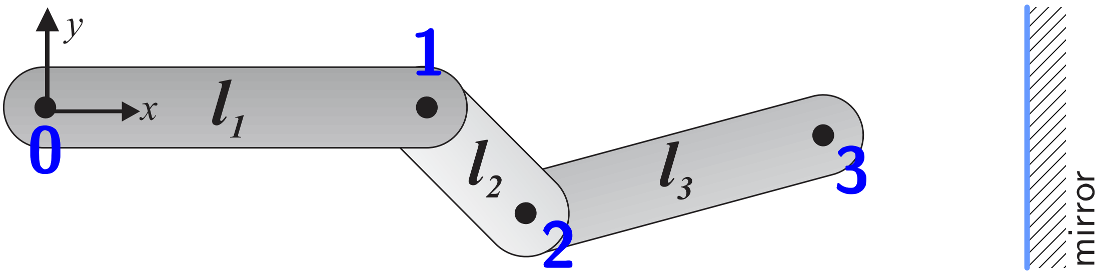

Figure 1: A 3R manipulator for cleaning a mirror

It has the following transformation matrices between frames:

$$
{}^{0}\mathbf{T}_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & l_1 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & l_1 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^{1}\mathbf{T}_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & l_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^{2}\mathbf{T}_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & l_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & l_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

(a) ***Spaces***

What are the **configuration space** and **workspace** for this robot arm?

(b) ***Transformation***

What is the overall **Transformation Matrix** (${}^{0}\boldsymbol{T}_3$) of the tip relative to the base (at {0})?

Continued Overleaf…

(c) ***Looking back on this***
What is the **Transformation Matrix** (${}^{3}\boldsymbol{T}_{\mathbf{0}}$) of the base relative to the tip?
[**hint:** if you are running short of time, please just outline the solution relative to ${}^{0}T_{3}$]

(d) ***Forward kinematics***
From this, what is the **Forward Kinematics** for this arm for any general three input angles $(\theta_1, \theta_2, \theta_3)$?

Continued Overleaf…

(e) ***Inverse kinematics***
What is the **Inverse Kinematics** for this problem for a general end-effector pose given by $p_e = (x_e, y_e, \phi)$?

(f) ***More than one way to get there***
How many real solutions will the Inverse Kinematics (from part (e)) have?
Will it always have this many solutions for all points p_e? If so, why? If not, why not?
And, in which case, what is a test for finding the number of real solutions?

Continued Overleaf…

(g) ***A new line of reflection* [EXTRA CREDIT]**
As noted, there is a mirror located to the right at a distance of **8λ** cm away from the base (frame {0}) of the robot.

If the robot lengths are $(l_1, l_2, l_3) = (4\lambda, 2\lambda, 3\lambda)$ respectively, then please give an expression for $(\theta_1, \theta_2, \theta_3)$ such that the arm is in contact with the mirror.
[i.e. if the arm were in contact with the mirror, what is the manifold of solutions?]

4. Robots in Wonderland…. [30 Points]

Hatter, the green tea robot, is going to have a tea party! To prevent a spill (motion), for one does not want to upset the Queen (of Hearts), it needs to place the teapot on a table with exact force.

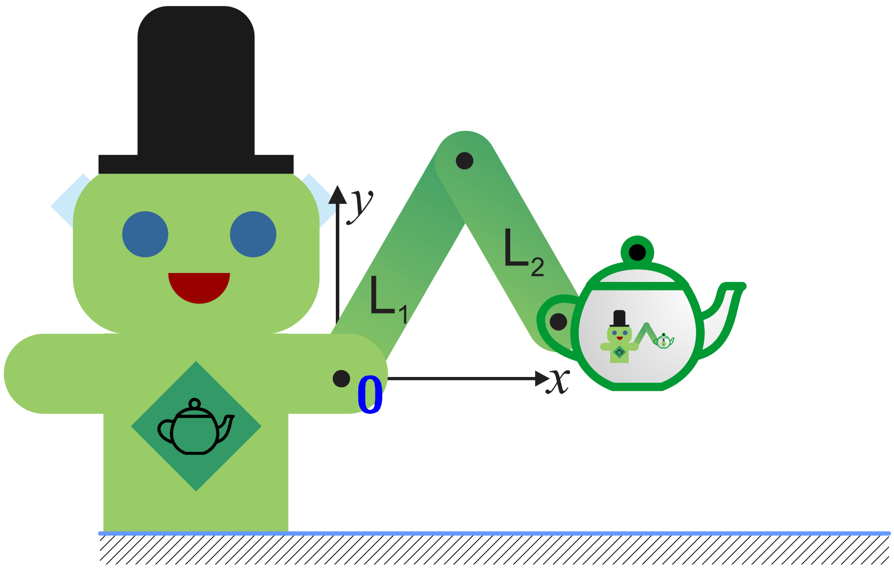

Figure 2: A manipulator needs to place the teapot on the table (crosshatch)
[PS. So as to serve tea to March Hare and Dormouse at 6:00pm ☺]

As shown in Figure 2, please assume:

- The first link, L_1, has mass m_1 and length 20 cm.
- The second link, L_2, has mass m_2 and length 15 cm.
- The teapot has mass $m_t = m_1 + m_2$ (m_t is not empty) and may be considered a vertically centred 10 cm × 10 cm box (i.e. from the end of L_2 to the teapot's bottom is 5 cm).
- The table is 30 cm below the base of the robot arm (i.e. $y_{table} = -30\ cm$ in frame {0}).

(a) ***Forward kinematics*** – Assuming the teapot angle can be controlled, what is the forward kinematics for this arm? [**Hint**: see also Question 3, part (d)]

Continued Overleaf…

(b) What is the Jacobian for this robot arm?

Continued Overleaf…

(c) If the teapot has to be placed down **flat** (at a pose of 0° relative to frame {0}) with precisely **10 N** force in the vertical (***y-axis***) direction with no force in the horizontal (***x-axis***) direction.
[1] What is the **reaction force** on the teapot?
[2] *And*, what is the resulting (additional) reaction **torques** of the contact on the robot joints?

5. Truth in Robotics! [10 Points]

Please state if the following statements are generally **TRUE (T)** or **FALSE (F)**

(Kindly circle the answer Ⓣ or Ⓕ, a brief justification may be *optionally* added below)

(a) **If** $A \in SE(n)$, **then** $det\ A = +1$ **[T | F]**

(b) A 3R3P manipulator must be redundant in 3D space **[T | F]**

(c) $R_x\left(\frac{\pi}{2}\right)R_y(\theta)\left(R_x\left(\frac{\pi}{2}\right)^T\right) = R_z(\theta)$ **[T | F]**

(d) All elements of a Jacobian have uniform units of measure **[T | F]**

(e) For a robot arm **with redundancies,** $\mathbf{det}(\boldsymbol{J}) = \mathbf{0}$ will give the singular poses. **[T | F]**

Extra Working Paper