⇒ Sort Pattern: **A** ⇐

METR 4202 -- Advanced Controls & Robotics

Individual Quiz **Solutions** September 23,2015

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1. Consider a general 2D transformation (e.g., for images) where **x'** is a point in the second image and **x̲** is a point in the first image.

   a. If we assume the transformation is **rigid (Euclidean)**, how many point pairs (i.e., **x'** and **x̲** together) are needed to recover the transformation?

   (Lecture 6, Slide 28)
   3 DOF, each point pair gives two constraints
   and you can't have 1.5 points, so→ ***2 points***

   b. If we assume the transformation is a **similarity transformation**, how many point pairs (i.e., **x'** and **x̲** together) are needed to recover the transformation?

   4 DOF → ***2 points***

   c. If we assume the transformation is an **affine transformation**, how many point pairs (i.e., **x'** and **x̲** together) are needed to recover the transformation?

   6 DOF → ***3 points***

2. What are the differences between the joint space, workspace and operation space? (explain fundamentally, not just their definitions).

The difference is with respect to what basis are we defining our robot's operation.
In other words, what states are fundamentally being described -- kinematically (internal and external, or sometimes denoted as proprioceptively and exterioceptively) and kinetically. The joint space is the space of all joint values (e.g., angles for revolute joints and positions for prismatic joints), the workspace is the space of points with respect the an external world or working set (typically the cartesian points that describe the end effector of the arm). The operation space is the set of points that describe the operations needed for the arm to complete the task and is typically a super-set of the workspace including the forces, etc.

3. What does the Null-space of the Jacobean inform or imply for a serial robot?

If the arm has redundant degrees of freedom, it informs the redundancy. That is places where joint motion will result in zero end-effector / workspace velocity

4. What is a distinguishing feature of SFM's operation, in particular over SLAM? (explain fundamentally, not just the definition).

SFM is a ***batch*** *(off-line, has all the data before it starts) process that uses a nonlinear optimization to correct the drift ("bundle adjustment"), whereas SLAM is an* ***online*** *estimation (recovers this as you go) of location and the map together and typically uses loop closure to provide constraints for cancelling drift.*

5. Consider two homogenous points $\mathbf{P}_1$ and $\mathbf{P}_2$. For parts (a), (b) and (c), what is the equation of the line between them?

   a. Two generic non-collinear points If $\mathbf{P}_1$= **(a, b, c)** and $\mathbf{P}_2$= **(d, e, f)**

   b. If $\mathbf{P}_1$= **(3, 2, 1)** and $\mathbf{P}_2$= **(2.5, 2, 1)**

   c. If $\mathbf{P}_1$= **(4, 2, 1)** and $\mathbf{P}_2$= **(0, 2, 1)**

   d. Are the lines found in part (b) and (c) parallel to each other? Briefly, why?

(Lecture 6, Slide 43) gives that the line $\mathbf{l}=P1\times P2$*. Starting with (c) first, we get L=[bf-ce, cd-af, ae-bd]. We then normalize as we prefer that the last terms is 1.*
(a) is (0, -½, 1) and (b) (0, -4, 8), which normalizes to (0, -½ ,1). They are parallel to each other as they are the same line.

6. Consider the camera calibration process with *planar* calibration object (e.g., a planar checkerboard). The object possesses **M** distinct features. The camera takes **K** images.

   You may assume an undistorted camera with following following characteristics:
   - Zero skew (orthogonal pixel arrangement)
   - Unity aspect ratio (square pixels)
   - Known image center (at ½ of image height and width)

   a. Given these assumptions, what unknown parameters are there to calibrate?
   (hint: what intrinsic parameters need to be recovered? And, what extrinsic parameters need to be recovered for each image?)

   In general there are 5 intrinsic parameters, namely: (f_x, f_y, s, o_x, o_y). In this case, the assumptions above imply: (a) s = 0 (b) f_x=f_y, and (c) $o_{x,}$ and o_y are known (o_x=w/2 and o_y=l/2). Thus, we have one value left, f_x or simply f.

   Also, the extrinsic parameters are 3 Euler angle orientations and 3 displacements {φ_i, γ_i, ψ_i, $T_{X,i}$, $T_{Y,i}$, $T_{Z,i}$}, for each image i.

   b. For the K images, each of M features, how many constraints are given?

   2KM (because each feature point has two pixel scalar values)

   c. For the K images, each of M features, how many parameters need to be calibrated?

   Continuing from (a) above…

   In addition there are are 2M parameters for the unknown feature location on the planar calibration object. This is because we know $Z_j^W = 0$ for all features j (because it's a planar object). So we add the object parameters {X_j^W , Y_j^W } to the set of free parameters. However, the definition of the object reference.
   frame is somewhat arbitrary.We can therefore define $X_1^W = Y_1^W = 0$, and $X_2^W = 0$.

   This reduces the number of recoverable parameters by 3. This definition constrains the location of all other features on the board.

   Thus, we have 1 + 6K + 2M − 3 = ***6K + 2M − 2*** *unknown parameters, with* ***K*** *being the number of images and* ***M*** *the number of features on the calibration pad.*

   d

d. For M = 4, what is the minimum number of images needed to determine the calibration?

We have to estimate 6K +2M − 2 parameters from 2KM constraints.

Hence we have:

$2KM \geq 6K + 2M - 2$

$\Rightarrow 2KM - 6K \geq 2M - 2$

$\Rightarrow K(2M - 6) \geq 2M - 2$

$\Rightarrow K \geq \frac{2M-2}{2M-6}$

For M=4 we get:

$\Rightarrow K \geq \frac{6}{2} \Rightarrow K \geq 3$

Thus, for 4 features, we need 3 images

7. Please state if the following statements are generally **TRUE (T)** or **FALSE (F)**

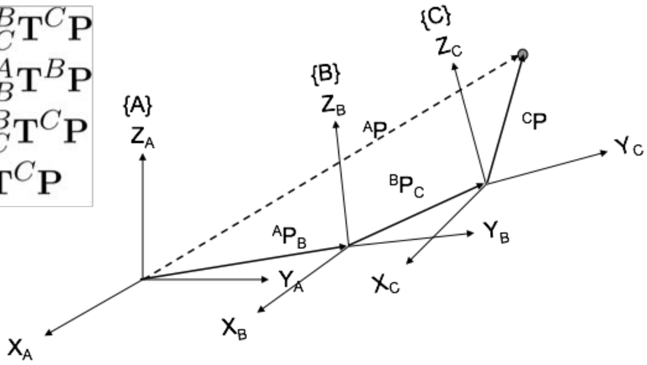
a.	The inverse of a rotation matrix is always its transpose.	**True**
b.	The inverse of a transformation matrix is its transpose.	*false*
c.	Homogeneous transforms are linear operations.	**True**
d.	In DH one of the four parameters (a, α, d, θ) must be 0.	*false*
e.	The inverse kinematics of a 6R arm is closed form with 16 solutions.	**True**
f.	Straight lines remain straight under a perspective transformation.	**True**
g.	For a manipulator, the torque needed is a function of the pose.	**True**
h.	RGB colour spaces are invariant to changes in illumination.	*false*
i.	Local perspective transformations are approximately affine transformations.	**True**
j.	The fundamental matrix is invertible.	*false*

8. What small changes in properties is SIFT invariant to?
    a. Rotation ✔
    b. Illumination ✔
    c. ~~Affine~~
    d. (a) and (b)
    e. (a) and (b) and (c)

9. For a general coordinate transformation between four frames {A}, {B}, {C}, and {D}. What is the overall transformation matrix ${}^{A}_{D}T$ between {A} and {D} as a function of the individual Rotations [${}^{i+1}_{i}R$, eg ${}^{a}_{b}R$] and Positions [${}^{i+1}_{i}P$, eg ${}^{a}_{b}P$].

(Lecture 2, Slide 41)

$$
\begin{aligned}
{}^{B}\mathbf{P} &= {}^{B}_{C}\mathbf{T}\,{}^{C}\mathbf{P} \\
{}^{A}\mathbf{P} &= {}^{A}_{B}\mathbf{T}\,{}^{B}\mathbf{P} \\
&= {}^{A}_{B}\mathbf{T}\,{}^{B}_{C}\mathbf{T}\,{}^{C}\mathbf{P} \\
&= {}^{A}_{C}\mathbf{T}\,{}^{C}\mathbf{P}
\end{aligned}
$$

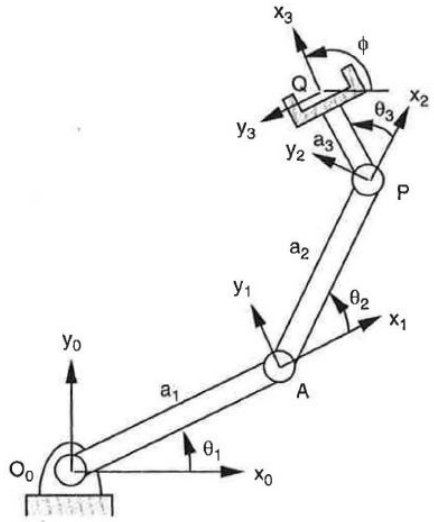


$$
{}^{A}_{C}\mathbf{T} = \begin{bmatrix} {}^{A}_{B}\mathbf{R}\,{}^{B}_{C}\mathbf{R} & {}^{A}\mathbf{P}_{B} + {}^{A}_{B}\mathbf{R}\,{}^{B}\mathbf{P}_{C} \\ 0\;\;0\;\;0 & 1 \end{bmatrix}
$$

Thus,

$$
T = \begin{bmatrix} {}^{a}_{b}R\;{}^{b}_{c}R\;{}^{c}_{d}R & {}^{a}_{b}P + {}^{a}_{b}R\,{}^{b}_{c}P + {}^{a}_{b}R\,{}^{b}_{c}R\,{}^{c}_{d}P \\ 0\;\;0\;\;0 & 1 \end{bmatrix}
$$

10. Consider the planar manipulator shown below.



Assume the following properties:

- $\mathbf{a_1}$ = 42 cm
- $\mathbf{a_2}$ = 32 cm
- $\mathbf{a_3}$ = 15 cm
- $\boldsymbol{\theta_1}$, $\boldsymbol{\theta_2}$ and $\boldsymbol{\theta_3}$ may take on any value from 0 to 360°

a. Provide two classes of pose where the arm will be at a singularity

- *Joint limits*
- *Point P aligns with the origin O_0 (or the y-axis)*

b. What value of joint angles will get the tip to the position [0, 89] cm?

[90°, 0°, 0°] , Found by inspections to notice that the point is at the end of the workspace (42+32+15 = 89)

c. What value of joint angles will get the tip to the position [21.4, 53] cm with an orientation of 180 degrees?

[30°, 60°, 90°]. Also found by inspection as the sum of the angles $\boldsymbol{\theta_1}+\boldsymbol{\theta_2}+\boldsymbol{\theta_3}=180°$. May be found by closed form inverse kinematics too!